Enhanced Decision Support for a Changing Electricity Landscape:

The GenX Configurable Electricity Resource Capacity Expansion Model

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Abstract

The electric power sector is currently undergoing several important transitions, which individually and collectively have the potential to transform the design, operation, and characteristics of electricity systems, including: decarbonization of electricity supplies; increased adoption of variable renewable energy and distributed energy resources; digitization of power systems; and electrification of greater shares of heating, transportation, and industry. In the face of these transformations, many conventional electricity resource capacity expansion models are no longer adequate for rigorous decision support and policy analysis. This working paper describes the formulation of "GenX," a highly-configurable electricity resource capacity expansion model that incorporates several state-of-the-art improvements in electricity system modeling to offer improved decision support for a changing electricity landscape. GenX is a constrained optimization model that determines the mix of electricity generation, storage, and demand-side resource investments and operational decisions to meet electricity demand in a future planning year at lowest cost subject to a variety of power system operational constraints and specified policy constraints, such as CO_2 emissions limits. The appropriate level of model resolution with regards to chronological variability of electricity demand and renewable energy availability, power system operational detail and unit commitment constraints, and transmission and distribution network representation each vary for a given planning problem or policy question. As such, the GenX model is designed to be highly configurable, with several different degrees of resolution possible on each of these three key dimensions. The model is capable of representing a full range of conventional and novel electricity resources, including thermal generators, variable renewable resources (wind and solar), run-of-river, reservoir and pumped-storage hydroelectric generators, energy storage devices, demand-side flexibility, and several advanced technologies such as high temperature nuclear reactors and carbon capture and storage. Two optional modules also allow modeling of heat storage, industrial heat demand, and co-generation of heat and power; and distributed energy resources deployed at distribution voltages. The model has been implemented in Julia Language.

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Contents

1 Introduction

The electric power sector is currently undergoing several important transitions, which individually and collectively have the potential to transform the design, operation, and characteristics of electricity systems. These transitions include the drive to decarbonize electricity generation to help confront climate change, increased adoption of distributed energy resources and decentralization of electricity service provision, digitization of power systems, electrification of greater shares of transportation, heating and industrial energy demand, and the growth of variable renewable energy resources such as wind and solar energy. These transitions are occurring against the backdrop of rapidly changing technology costs, introduction of new technologies and resources, and significant policy uncertainty.

Decision support tools, including power system optimization models, can help explore these important transitions, illuminate key mechanisms, uncertainties and risks, and help guide power system planners, policy makers and businesses. In particular, capacity expansion (or capacity planning) modeling tools have historically been used to help explore the least-cost mix of various available electricity generation resources under a given scenario. Capacity expansion models are heavily employed in least-cost or integrated resource planning [1–4] for regulated or publicly-owned utilities. In addition, while electricity markets in many jurisdictions are now competitive and investment decisions are made by diverse individual actors, the electricity sector remains heavily regulated everywhere and extensively influenced by market design decisions, regulated tariffs, and public policy incentives (e.g., tax policies, mandates). Capacity expansion models can thus play a critical role in policy analysis and indicative planning [5] to inform the many processes by which regulators and states guide electricity market actors to achieve societal objectives, including economic efficiency, security, and environmental outcomes. Capacity expansion models also serve a useful role in techno-economic assessment of emerging electricity generation, storage, and demand-side resources and their impact on electricity systems (e.g., [6–9]).

Unfortunately, conventional capacity expansion planning methods are no longer adequate to rigorously evaluate options in the face of the ongoing transformation of power systems. Conventional methods, including the "screening curves" or non-sequential load duration curve methods [2, 10–13] at the heart of many widely used linear programming-based capacity expansion models (e.g., [14–16]), typically ignore several features that are increasingly important to electricity resource investment and planning decisions.

- First, conventional models typically represent the operation of power systems with very limited detail, frequently ignoring key features such as inter-temporal constraints on rates of change in power plant output (ramp rates), unit commitment decisions (start-up, shut-down) and minimum stable output levels for thermal power plants, and the various classes of frequency regulation and operating reserves required to ensure reliability in the face of unplanned power plant or transmission line outages or errors in electricity demand or renewable energy forecasts. Abstracting these important power system operational details in capacity planning models can result in significant errors, particularly in power systems with high shares of variable renewable energy resources.

- Second, many models represent chronological variability of electricity demand and renewable energy resource quality in a highly abstracted manner, such as a limited number of "time slices," "load blocks," or simplified "load duration curves." Failing to accurately account for the chronological variability and correlations between both renewable energy output and electricity demand can also result in significant errors in capacity expansion decisions.

- Third, conventional models typically ignore electricity transmission and distribution network infrastructure. This prevents accurate evaluation of the potential value and impact of distributed energy resources, which are deployed at different voltage levels and locations in the distribution network, as well as renewable energy resources, which entail tradeoffs between siting at locations with differing resource quality and differing impact on transmission networks or requirements for network expansion.

- Fourth, most power system models represent electricity demand as exogenous and include little if any integration with other adjacent sectors, such as transportation or heating energy demand. As demand-side flexibility becomes an increasingly important component of power system planning and operations and as electricity expands to provide a greater share of final energy demand in transportation, heating, and industrial processes, capacity planning models will need to be further improved to capture these important linkages.

In short, new decision support tools are needed for a changing electricity landscape, including improved electricity resource capacity expansion models. Recent research has improved capacity planning methods and begun to address many of the shortcomings described above, including: enhanced representation of operation and unit commitment constraints in capacity planning models [17–23]; representation of transmission network expansion and constraints and tradeoffs in renewable energy siting [21, 24–28]; improved representation of demand-side flexibility and distributed energy resources [29]; and new methods for representing temporal variability in demand and renewable resource quality [20, 22, 30–33].

Despite these methodological developments and ever-improving computational resources, electricity resource capacity expansion models based on linear programming (LP) or mixed integer linear programming (MILP) methods continue to be constrained by computational complexity and the high dimensionality of realistic power system optimization problems. These computational constraints lead to inevitable tradeoffs between computational tractability and the degree of model resolution on each of three key dimensions:

1. Representation of chronological variability in demand and renewable resources;

2. Representation of power system operational detail and constraints; and

3. Representation of transmission and distribution networks.

Figure 1 depicts several common options for the level of detail along each of these three dimensions. Additional factors adding to dimensionality include: the number of distinct resource types and options considered; modeling linkages to other sectors, such as heating and transportation demand; and endogenous treatment of uncertainty via stochastic or robust optimization methods. Unfortunately, it remains infeasible to simultaneously model the highest degree of detail possible along all dimensions at once. Instead, power system modelers must carefully select the appropriate level of detail or abstraction and adopt "dimensionality reduction techniques" appropriate to the questions at hand.

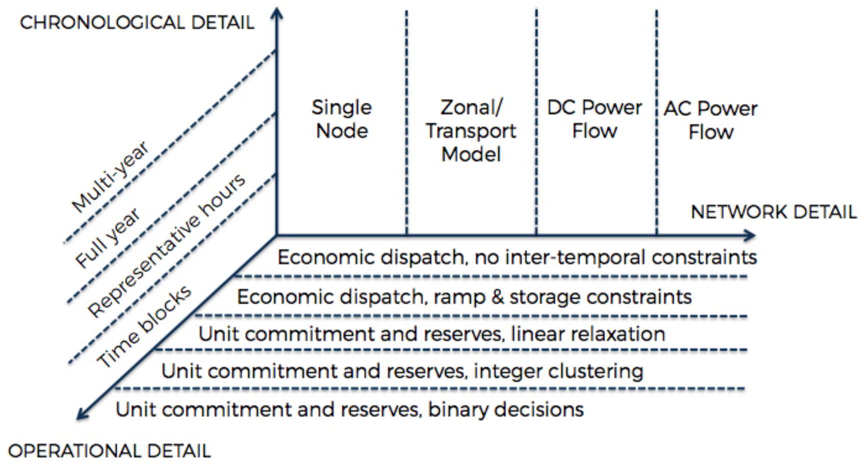

Figure 1: Common options for model resolution along three key dimensions

In light of these challenges, this paper introduces "GenX," a highly configurable electricity resource capacity expansion model intended to offer improved decision support capabilities for a changing electricity landscape. This working paper describes the current version of the model and is intended as a "living document" to be revised upon subsequent improvements and iterations of the model.

GenX is a constrained optimization model that determines the mix of electricity generation, storage, and demand-side resource investments and operational decisions to meet electricity demand in a future planning year at lowest cost subject to a variety of power system operational constraints and specified policy constraints, such as CO_2 emissions limits. As the appropriate level of model resolution with regards to chronological variability of load and renewable energy availability, operational detail and constraints, and transmission and distribution network representation varies for a given planning problem or policy question, the model is designed to be highly configurable, with several different degrees of resolution possible on these three key dimensions. These different configurations include a variety of state-of-the-art techniques for addressing each of the three dimensions described.

The remainder of this document is structured as follows: Section 2 describes the general structure of the GenX model, including the various options for configuring model resolution with regards to chronology, operational detail, and networks. Section 3 lists the notation used throughout the remainder of the document. Section 4 provides the full mathematical formulation of the model. Section 5 provides a comprehensive description of the GenX model including options for configuring the model. Finally, Appendix A lists a set of publications (theses, working papers, reports, and published journal articles) employing GenX, to give the reader an idea of the various use cases for the model.

2 Model Introduction

The GenX model formulation (described in detail in subsequent sections) allows for the simultaneous co-optimization of seven interlinked power system decision layers:

1. Capacity expansion planning (e.g., investment and retirement decisions for a full range of centralized and distributed generation, storage, and demand-side resources)
2. Hourly dispatch of generation, storage, and demand-side resources,
3. Unit commitment decisions and operational constraints for thermal generators,
4. Commitment of generation, storage, and demand-side capacity to meet system operating reserves requirements,
5. Transmission network power flows (including losses) and network expansion decisions,
6. Distribution network power flows, losses, and network reinforcement decisions, and
7. Interactions between electricity and heat markets.

Depending on the dimensionality of the problem, it may not be possible to model all seven decision layers at the highest possible resolution of detail, so the GenX model is designed to be highly configurable, allowing the user to specify the level of detail or abstraction along each of these seven layers or to omit one or more layers from consideration entirely.

For example, while investment and dispatch decisions (Layers 1 and 2) are a consistent feature of the model under all configurations, the user has several options with regards to representing the operational constraints on various thermal power plants (e.g., coal, gas, nuclear, and biomass generators). Unit commitment (e.g., start-up and shut-down) decisions [34] (Layer 3) can be modeled at the individual power plant level (as per [20]); by using an efficient clustering of similar or identical units (as per [17–19]); by using a linear relaxation (or convex hull) of the integer unit commitment constraints set; or ignoring unit commitment decisions entirely and treating generator output as fully continuous. Furthermore, different levels of resolution can be selected for each individual resource type, as desired (e.g., larger thermal units can be represented with integer unit commitment decisions while smaller units can be treated as fully continuous). In such a manner, the model can be configured to represent operating constraints on thermal generators at a level of resolution that achieves a desired balance between abstraction error and computational tractability and provides sufficient accuracy to generate insights for the problem at hand.

The model can also be configured to consider commitment of capacity to supply frequency regulation and operating reserves needed by system operators to robustly resolve short-term uncertainty in load and renewable energy forecasts and power plant or transmission network failures (Layer 4). Alternatively, reserve commitments can be ignored if desired.

Similarly, the model allows for transmission networks to be represented at several levels of detail (Layer 5) including: at a nodal or zonal level with a linearized DC approximation of AC power flows between nodes or zones (as per [28, 29, 35]); at a zonal level with transport constraints on power flows between zones (as per [25, 26, 36]); or as a single zone problem where transmission constraints and flows are ignored. In cases where a nodal or zonal transmission model is employed, network capacity expansion decisions can be modeled or ignored, and transmission losses can be represented with a piecewise linear approximation of quadratic losses due to power flows between nodes or zones (as per [37, 38]), with the number of segments in the piecewise approximation specified by the user as desired. In a multi-zonal or nodal configuration, GenX can therefore consider siting generators in different locations, including balancing tradeoffs between access to different renewable resource quality, siting restrictions, and impacts on network congestions, power flows and losses.

If desired, GenX can also represent distribution networks using a novel zonal approximation (Layer 6, see Section 5.17), with each zone representative of a different distribution network topology and voltage level (see Figure 2). Electricity demand as well as capacity investment and operational decisions are then indexed across each zone in the system, enabling the model to select the optimal location of capacity investments and operations in each location, including considering distributed energy resources (DERs, such as distributed solar PV, energy storage, fuel cells, etc.) in different distribution voltage zones. Power flows between voltage levels can be constrained to represent transformer capacity constraints. Losses due to power flows within each distribution network zone can be represented as a piecewise linear approximation of a polynomial function of power injections and withdrawals within each zone, which can be parameterized based on detailed offline AC power flow simulations for the representative networks [39]. Distribution network reinforcement costs associated with changes in peak power injections or withdrawals within each distribution voltage level and network zone can also be represented as piecewise linear functions parameterized based on offline modeling using a suitable combination of network planning and optimal power flow modeling [40]. Thus, the GenX model can be configured to consider DERs, including balancing tradeoffs between economies of unit scale at different voltage levels on the one hand, and the differential impacts or benefits of locating DERs at different zones or voltage levels on the other hand.

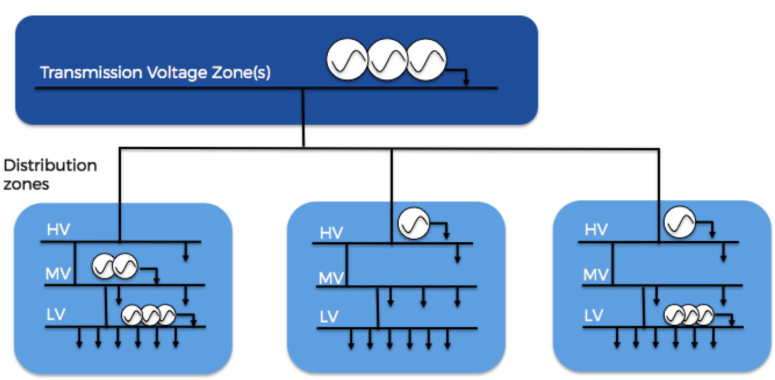

Figure 2: Schematic representation of multi-zonal configuration with transmission and distribution network zones and siting of resources at different voltage levels and zones

Finally, the model can be configured to consider interactions between electricity generation and the heating sector (Layer 7), including modeling combined heat and power generation, use of excess electricity for electrically-heated thermal storage or supply of industrial process heat, or use of heating inputs for topping cycles in advanced generators such as a Nuclear Air-Brayton Combined Cycle (NACC) concept [9, 41].

With appropriate configuration of the model, GenX thus allows the user to tractably consider several interlinking decision layers in a single, monolithic optimization problem that would otherwise have been necessary to solve in different separated stages or models. Figure 3 reflects the range of configurations currently possible along the three key dimensions of chronological detail, operational detail, and network detail.

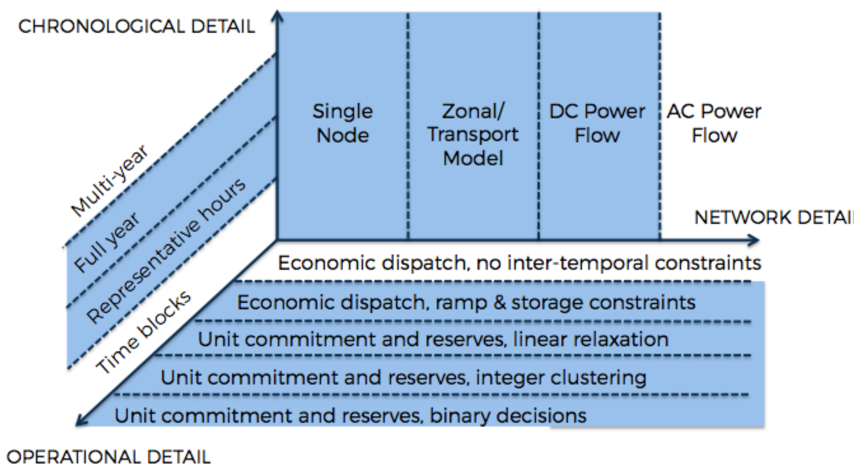

Figure 3: Range of configurations currently implemented in GenX along three key dimensions of model resolution

The model is usually configured to consider a full year of operating decisions at an hourly interval to represent some future planning year. In this sense, the current formulation is *static* because its objective is not to determine when investments should take place over time, but rather to produce a snapshot of the minimum-cost generation capacity mix under some pre-specified future conditions. However, the current implementation of the model can be run in sequence (with outputs from one planning year used as inputs for another subsequent planning year) to represent a step-wise or myopic expansion of the network. In the future, the model may be revised to allow simultaneous co-optimization of sequential planning decisions. In addition, to improve computational tractability, the model can consider a reduced number of representative hours within the future planning year, selected using an appropriate time domain reduction technique [20, 22, 30–33].

From a centralized planning perspective, this formulation can help to determine the investments needed to supply future electricity demand at minimum cost, as is common in least-cost utility planning or integrated resource planning processes. In the context of liberalized markets, the model can be used by regulators and policy makers for indicative energy planning or policy analysis in order to establish a long-term vision of efficient market and policy outcomes. The model can also be used for techno-economic assessment of emerging electricity generation, storage, and demand-side resources and to enumerate the effect of parametric uncertainty (e.g., technology costs, fuel costs, demand, policy decisions) on the system-wide value or role of different resources.

The high level structure of the GenX model is presented in Table 1, with reference to each equation implementing the various components of the configurable model. GenX is implemented using Julia Language [42]. Julia is a high-level, high-performance dynamic programming language for technical computing, with syntax that is familiar to users of other technical computing environments. Additionally, the model is implemented using JuMP [43], a domain-specific modeling language for mathematical programming embedded in Julia. JuMP supports a number of open-source and commercial solvers for a variety of problem classes, including linear programming, mixed-integer programming, second-order conic programming, semidefinite programming, and nonlinear programming. The solver used in the present work was Gurobi developed by Gurobi Optimization [44].

Table 1: Model's General Structure and Reference to Equations

Minimize Investment costs + Operational costs - Heat Market Sales	(1)
subject to:	
Investment decisions constraints	(2–3)
CO_2 emissions constraints	(4–5)
Minimum renewable energy mandate constraints	(6–7)
Demand balance constraint	(8)
Transmission network related constraints	(15–29)
Unit commitment constraints	(30–39)
Constraints for thermal technologies w/unit commitment	(36–35)
Constraints for other thermal	(40–43)
Renewable technologies operational constraints	(44–45)
Storage resources operational constraints	(46–52)
Demand-side management constraints	(53–55)
Demand response constraint	(56)
NACC operational constraints	(57–60)
Heat storage operational constraints	(61–69)
Hydro reservoir resources operational constraints	(70–74)
Operating reserves related constraints	(76–117)
Distribution network related constraints	(118–136)
Non-negativity/integrality constraints	(137–160)

3 Notation

3.1 Model Indices and Sets

Table 2: Model Indices and Sets

Notation	Description
$t, e \in \mathcal{T}$	where t denotes an hour and $\mathcal{T}$ is the set of hours in the data series (e is an alternate index).
$z, d \in \mathcal{Z}$	where z denotes a zone/node and $\mathcal{Z}$ is the set of zones/buses in the network (d is an alternate index).
$l \in \mathcal{L}$	where l denotes a line and $\mathcal{L}$ is the set of transmission lines in the network.
$y, x \in \mathcal{G}$	where y denotes a technology and $\mathcal{G}$ is the set of available technologies (x is an alternate index).
$s \in \mathcal{S}$	where s denotes a segment and $\mathcal{S}$ is the set of consumers segments for price-responsive demand curtailment.
$m \in \mathcal{M}$	where m denotes a segment used in piecewise approximation of quadratic functions and $\mathcal{M}$ is the set of segments $[0 : M]$
$\mathcal{H} \subset \mathcal{G}$	where $\mathcal{H}$ is the subset of thermal resources.
$\mathcal{RE} \subset \mathcal{G}$	where $\mathcal{RE}$ is the subset of renewable energy resources.
$\mathcal{O} \subset \mathcal{G}$	where $\mathcal{O}$ is the subset of storage resources excluding heat storage.
$\mathcal{DR} \subset \mathcal{G}$	where $\mathcal{DR}$ is the subset of demand response resources.
$\mathcal{AN} \subset \mathcal{G}$	where $\mathcal{AN}$ is the subset of advance nuclear resources (NACC).
$\mathcal{HO} \subset \mathcal{G}$	where $\mathcal{HO}$ is the subset of heat storage resources.
$\mathcal{W} \subset \mathcal{G}$	where $\mathcal{W}$ is the subset of hydro reservoir resources.
$\mathcal{UC} \subset \mathcal{H}$	where $\mathcal{UC}$ is the subset of thermal resources subject to unit commitment constraints.
$\mathcal{D} \subset \mathcal{RE}$	where $\mathcal{D}$ is the subset of dispatchable[a] renewable resources.
$\mathcal{ND} \subset \mathcal{RE}$	where $\mathcal{ND}$ is the subset of non-dispatchable[b] renewable resources.
$\mathcal{E} \subset \mathcal{L}$	Subset of transmission lines lines eligible for reinforcement
$\mathcal{R} \subset \mathcal{Z}$	Subset of transmission zones
$\mathcal{V} \subset \mathcal{Z}$	Subset of distribution zones
$\mathcal{PW} \subset \mathcal{T}$	Subset of hours in which aggregate peak withdrawal may occur in distribution zones
$\mathcal{PI} \subset \mathcal{T}$	Subset of hours in which aggregate peak injection may occur in distribution zones

[a]Generation curtailment allowed i.e., utility scale
[b]Generation curtailment not allowed i.e., residential

3.2 Decision Variables

Table 3: Decision Variables

Notation	Description
$\Omega_{y,z} \in \mathbb{R}_+$	Installed capacity of technology y in zone z [MW].
$\Delta_{y,z} \in \mathbb{R}_+$	Retired capacity of technology y from existing capacity in zone z [MW].
$\Theta_{y,t,z} \in \mathbb{R}_+$	Energy injected into the grid by technology y at hour t in zone z [MWh].
$\Pi_{y,t,z} \in \mathbb{R}_+$	Energy withdrawn from grid by technology y at hour t in zone z [MWh].
$\Gamma_{y,t,z} \in \mathbb{R}_+$	Stored energy level of technology y at end of hour t in zone z [MWh].
$\Lambda_{s,t,z} \in \mathbb{R}_+$	Non-served energy/curtailed demand from the price-responsive demand segment s at hour t in zone z [MWh].
$\ell_{l,t} \in \mathbb{R}$	Losses in line l at hour t [MWh].
$\Phi_{l,t} \in \mathbb{R}$	Power flow in line l at hour t [MWh].
$\triangle\varphi_l^{max} \in \mathbb{R}_+$	Expansion of transmission capacity in line l [MW].
$\Phi^+_{l,t}, \Phi^-_{l,t} \in \mathbb{R}_+$	Power flow absolute value auxiliary variables for line l [MW] at time t in positive (+) and negative (-) domains.
$\S^+_{m,l,t}, \S^-_{m,l,t} \in \mathbb{R}_+$	Segment m of piecewise approximation of quadratic transmission losses function for line l at time t [MW] in positive (+) and negative (-) domains.
$ON^+_{m,l,t}, ON^-_{m,l,t} \in \{0,1\}$	Activation variable for segment m of piecewise approximation of quadratic transmission losses function for line l at time t in positive (+) and negative (-) domains.
$\vartheta_{z,t} \in \mathbb{R}$	Bus angle of zone/bus z at hour t [rad].
$\upsilon_{y,t,z} \in \mathbb{Z}_+$	Commitment state of generator cluster y at hour t in zone z.
$\chi_{y,t,z} \in \mathbb{Z}_+$	Startup events of generator cluster y at hour t in zone z.
$\zeta_{y,t,z} \in \mathbb{Z}_+$	Shutdown events of generator cluster y at hour t in zone z.
$\epsilon_{y,t,z} \in \mathbb{R}_+$	Heat from storage sold by technology y at hour t in zone z [MWh].
$\sigma_{y,t,z} \in \mathbb{R}_+$	Heat from storage used for generation by technology y at hour t in zone z [MWh].
$\nu_{y,t,z} \in \mathbb{R}_+$	Heat from natural gas combustion used for generation by technology y at hour t in zone z [MWh].
$r^+_{y,z,t} \in \mathbb{R}_+$	Reserves contribution up [MW] from technology y in zone z at time t.
$r^-_{y,z,t} \in \mathbb{R}_+$	Reserves contribution down [MW] from technology y in zone z at time t.
$f^+_{y,z,t} \in \mathbb{R}_+$	Frequency regulation contribution up [MW] from technology y in zone z at time t.
$f^-_{y,z,t} \in \mathbb{R}_+$	Frequency regulation contribution down [MW] from technology y in zone z at time t.
$r^{+C}_{y,z,t} \in \mathbb{R}_+$	Reserves contribution up [MW] from storage technology y during charging process in zone z at time t.

Continued on next page

Table 3—continued from previous page

Notation	Description
$r^{-C}_{y,z,t} \in \mathbb{R}_+$	Reserves contribution down [MW] from storage technology y during charging process in zone z at time t.
$f^{+C}_{y,z,t} \in \mathbb{R}_+$	Frequency regulation contribution up [MW] from storage technology y during charging process in zone z at time t.
$f^{-C}_{y,z,t} \in \mathbb{R}_+$	Frequency regulation contribution down [MW] from storage technology y during charging process in zone z at time t.
$r^{+D}_{y,z,t} \in \mathbb{R}_+$	Reserves contribution up [MW] from storage technology y during discharging process in zone z at time t.
$r^{-D}_{y,z,t} \in \mathbb{R}_+$	Reserves contribution down [MW] from storage technology y during discharging process in zone z at time t.
$f^{+D}_{y,z,t} \in \mathbb{R}_+$	Frequency regulation contribution up [MW] from storage technology y during discharging process in zone z at time t.
$f^{-D}_{y,z,t} \in \mathbb{R}_+$	Frequency regulation contribution down [MW] from storage technology y during discharging process in zone z at time t.
$r^{+,unmet}_{t} \in \mathbb{R}_+$	Unmet reserves up [MW] in time t.
$r^{-,unmet}_{t} \in \mathbb{R}_+$	Unmet reserves down [MW] in time t
$\ell_{z,t} \in \mathbb{R}+$	Losses within distribution zone z at hour t [MWh].
$\S^{+}_{m,z,t}, \S^{-}_{m,z,t} \in \mathbb{R}_+$	Segment m for piecewise approximation of quadratic term in distribution losses function for zone z at time t [MW] in the positive (+) and negative (-) domains.
$ON^{+}_{m,z,t}, ON^{-}_{m,z,t} \in \{0,1\}$	Activation variable for segment m of piecewise approximation of quadratic term in distribution losses function for zone z at time t in positive (+) and negative (-) domains.
$\triangle\lambda^{W}_{z} \in \mathbb{R}+$	New power withdrawal network capacity added to distribution zone z [MW].
$\triangle\lambda^{I}_{z} \in \mathbb{R}+$	New power injection network capacity added to distribution zone z [MW].
$\phi^{W}_{z,t} \in \mathbb{R}+$	Power withdrawal margin gained via optimal dispatch of distributed resources in distribution zone z [MW] at hour t.
$\phi^{I}_{z,t} \in \mathbb{R}+$	Power injection margin gained via optimal dispatch of distributed resources in distribution zone z [MW] at hour t.
$\S^{\phi,W}_{m,z,t} \in \mathbb{R}+$	Segment m for linear approximation of network withdrawal margin gained from optimal dispatch of distributed resources in zone z at hour t.

3.3 Parameters

Table 4: Model Parameters

Notation	Description
$D_{t,z}$	Electricity demand at hour t in zone z [MWh].
n_s^{slope}	Cost of non-served energy/demand curtailment for price-responsive demand segment s [$/MWh].
n_s^{size}	Size of price-responsive demand segment s as a fraction of the hourly zonal demand [%].
$\overline{\Omega_{y,z}}$	Maximum new capacity of technology y in zone z [MW].
$\overline{\Delta_{y,z}}$	Existing installed capacity of technology y in zone z [MW].
$\Omega_{y,z}^{size}$	Unit size of technology y in zone z [MW].
$\pi_{y,z}^{INVEST}$	Investment cost (annual amortization of total construction cost) for technology y in zone z [$/MW-yr].
$\pi_{y,z}^{FOM}$	Fixed O&M cost of technology y in zone z [$/MW-yr].
$\pi_{y,z}^{VOM}$	Variable O&M cost of technology y in zone z [$/MWh].
$\pi_{y,z}^{FUEL}$	Fuel cost of technology y in zone z [$/MWh].
$\pi_{y,z}^{START}$	Startup cost of technology y in zone z [$/startup].
$\epsilon_{y,z}^{CO_2}$	CO_2 emissions per unit energy produced by technology y in zone z [tons/MWh].
$\rho_{y,z}^{min}$	Minimum stable power output per unit of installed capacity for technology y in zone z [%].
$\eta_{y,z}^{loss}$	Self discharge rate per hour per unit of installed capacity for storage technology y in zone z [%].
$\eta_{y,z}^{up}$	Single-trip efficiency of storage charging/demand deferral for technology y in zone z [%].
$\eta_{y,z}^{down}$	Single-trip efficiency of storage discharging/demand satisfaction for technology y in zone z [%].
$\mu_{y,z}^{stor}$	Power to energy ratio of storage technology y in zone z [MW/MWh].
$\mu_{y,z}^{DSM}$	Maximum percentage of hourly demand that can be shifted by technology y in zone z [%].
$\overline{\tau_{y,z}}$	Time periods over which demand can be deferred using demand-side management technology y in zone z before demand must be satisfied [hours].
$\kappa_{y,z}^{up}$	Maximum ramp-up rate per time step as percentage of installed capacity of technology y in zone z [%/hr].
$\kappa_{y,z}^{down}$	Maximum ramp-down rate per time step as percentage of installed capacity of technology y in zone z [%/hr].
$\tau_{y,z}^{up}$	Minimum uptime for thermal generator type y in zone z before new shutdown [hours].
$\tau_{y,z}^{down}$	Minimum downtime or thermal generator type y in zone z before new restart [hours].
$\eta_{y,z}^{heat}$	Heat to electricity conversion efficiency for NACC technology y in zone z [%].

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Table 4—continued from previous page

Notation	Description
$\mu_{y,z}^{heat}$	Peak to base generation ratio of for NACC technology y in zone z [%].
$\rho_{y,t,z}^{max}$	Maximum available generation per unit of installed capacity during hour t for technology y in zone z [%].
$H_{t,z}$	Heat demand at hour t in zone z [MWh].
π_z^{HEAT}	Heat price in zone z [$/MWh].
$w_{y,z}^{level}$	Initial level of hydro reservoir y in zone z [%].
$\varphi_{l,z}^{map}$	Topology of the network, for line l: $\varphi_{l,z}^{map} = 1$ for zone z of origin, -1 for zone z of destination, 0 otherwise.
φ_l^{max}	Transmission capacity of line l [MW].
φ_l^{volt}	Transmission voltage of line l [kV].
φ_l^{ohm}	Transmission resistance of line l [Ohms].
φ_l^{loss}	Linear transmission losses per unit of power flow across line l [p.u.].
φ_l^{θ}	Maximum angle difference of line l [rad].
$\overline{\triangle\varphi_l^{max}}$	Maximum power flow capacity reinforcement for line l [MW]
π_l^{TCAP}	Transmission power flow reinforcement cost for line l [$/MW].
ϵ_z^{max}	CO_2 emissions constraint for zone z [tons/MWh].
$\gamma_{y,z}^{+}$	Max. contribution of capacity to reserves up [p.u.] for technology y in zone z
$\gamma_{y,z}^{-}$	Max. contribution of capacity to reserves down [p.u.] for technology y in zone z
$\iota_{y,z}^{+}$	Max. contribution to frequency regulation up [p.u.] for technology y in zone z
$\iota_{y,z}^{-}$	Max. contribution to frequency regulation down [p.u.] for technology y in zone z
R^{+D}	Reserves requirement up as a function of hourly load [%].
R^{+VRE}	Reserves requirement up as a function of hourly variable renewable resource availability[%].
R^{-D}	Reserves requirement down as a function of hourly load [%].
R^{-VRE}	Reserves requirement down as a function of hourly variable renewable resource availability[%].
F^{D}	Frequency regulation requirement as a function of hourly load [%].
F^{VRE}	Frequency regulation requirement as a function of hourly variable renewable resource availability[%].
π^{unmet}	Penalty for unmet reserve requirement [$/MW].
μ_z^{RE}	Minimum penetration of qualifying renewable energy resources required in zone z [%].
π_z^{DCAP}	Distribution network reinforcement cost for zone z [$/MW].
λ_z^{I}	Maximum aggregate power injection possible in distribution zone z [MW]
λ_z^{W}	Maximum aggregate power withdrawal possible in distribution zone z [MW]
$\overline{\triangle\lambda_z^{I}}$	Maximum distribution network aggregate injection capacity reinforcement for zone z [MW]
$\overline{\triangle\lambda_z^{W}}$	Maximum distribution network aggregate withdrawal capacity reinforcement for zone z [MW]

Continued on next page

Table 4—continued from previous page

Notation	Description
$N_z^{W,sqrt}$	Coefficient for square-root term for distribution network withdrawal margin gained via dispatch of distributed resources
$N_z^{W,linear}$	Coefficient for linear term for distribution network withdrawal margin gained via dispatch of distributed resources
$\overline{\phi_z^W}$	Maximum possible distribution network withdrawal margin that can be gained via dispatch of distributed resources
$S_{m,z}^{\phi,W}$	Slope of each segment m for linear approximation of square root term in function for network withdrawal margin that can be gained via dispatch of distributed resources [MW]
$\varphi_{z,d}^{down}$	Set of distribution zones d "downstream" of each distribution zone z: $\varphi_{z,d}^{down} = 1$ for zone d if $z = d$ or if there is a path from z to d and d is at a lower voltage than z, 0 otherwise.
φ_z^{Net}	Within zone distribution loss coefficient for quadratic term of polynomial function for losses due to net withdrawals in zone z
φ_z^W	Within zone distribution loss coefficient for linear term of polynomial function for losses due to aggregate withdrawals in zone z
φ_z^I	Within zone distribution loss coefficient for linear term of polynomial function for losses due to aggregate injections in zone z
φ_z^{Int}	Within zone distribution loss intercept coefficient for polynomial function for losses in zone z
M	Number of segment to use in piecewise linear approximation of quadratic terms in transmission and distribution losses functions

4 Model Formulation

This section presents the full mathematical formulation of the different equations that comprise the optimization model, starting with the objective function Eq. 1 and then all the different constraints Eq. 2–165. A textual description of each element of the model is provided in Section 5.

$$
\begin{aligned}
\min \Bigg\{ & \textstyle\sum_{z\in\mathcal{Z}}\sum_{y\in\mathcal{G}}\Big((\pi^{INVEST}_{y,z}\times\Omega^{size}_{y,z}\times\Omega_{y,z})+(\pi^{FOM}_{y,z}\times\Omega^{size}_{y,z}\times(\overline{\Delta_{y,z}}+\Omega_{y,z}-\Delta_{y,z}))\Big)\\
&+\textstyle\sum_{z\in\mathcal{Z}}\sum_{y\in\mathcal{G}}\sum_{t\in\mathcal{T}}\Big(((\pi^{VOM}_{y,z}+\pi^{FUEL}_{y,z})\times\Theta_{y,t,z})+(\pi^{VOM}_{y,z}\times\Pi_{y,t,z})+(\pi^{HEAT}_{z}\times\nu_{y,t,z})\Big)\\
&+\textstyle\sum_{z\in\mathcal{Z}}\sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}}\Big(n^{slope}_{s}\times\Lambda_{s,t,z}\Big)\\
&+\textstyle\sum_{z\in\mathcal{Z}}\sum_{y\in\mathcal{G}}\sum_{t\in\mathcal{T}}\Big(\pi^{START}_{y,z}\times\chi_{y,t}\Big)\\
&-\textstyle\sum_{z\in\mathcal{Z}}\sum_{y\in\mathcal{G}}\sum_{t\in\mathcal{T}}\Big(\pi^{HEAT}_{z}\times\epsilon_{y,t,z}\Big)\\
&+\textstyle\sum_{t\in\mathcal{T}}\Big(\pi^{unmet}(r^{+,unmet}_{t}+r^{-,unmet}_{t})\Big)\\
&+\textstyle\sum_{l\in\mathcal{L}}\Big(\pi^{TCAP}_{l}\times\triangle\varphi^{max}_{l}\Big)\\
&+\textstyle\sum_{z\in\mathcal{V}}\Big(\pi^{DCAP}_{z}\times(\triangle\lambda^{W}_{z}+\triangle\lambda^{I}_{z})\Big)\Bigg\}
\end{aligned}
\tag{1}
$$

subject to

$$\Omega_{y,z}\leq\frac{\overline{\Omega_{y,z}}}{\Omega^{size}_{y,z}}, \qquad \forall y\in\mathcal{G},\forall z\in\mathcal{Z} \tag{2}$$

$$\Delta_{y,z}\leq\frac{\overline{\Delta_{y,z}}}{\Omega^{size}_{y,z}}, \qquad \forall y\in\mathcal{G},\forall z\in\mathcal{Z} \tag{3}$$

$$\textstyle\sum_{y\in\mathcal{G}}\sum_{t\in\mathcal{T}}(\epsilon^{CO_2}_{y,z}\times(\Theta_{y,t,z}+\Pi_{y,t,z}))\leq\epsilon^{max}_{z}\times\sum_{t\in\mathcal{T}}D_{t,z}, \qquad \forall z\in\mathcal{Z} \tag{4}$$

$$
\begin{aligned}
&\textstyle\sum_{z\in\mathcal{Z}}\sum_{y\in\mathcal{G}}\sum_{t\in\mathcal{T}}(\epsilon^{CO_2}_{y,z}\times(\Theta_{y,t,z}+\Pi_{y,t,z}))\\
&\quad\leq\textstyle\sum_{z\in\mathcal{Z}}\sum_{t\in\mathcal{T}}(\epsilon^{max}_{z}\times D_{t,z}),
\end{aligned}
\tag{5}
$$

$$\textstyle\sum_{y\in\mathcal{RE}}\sum_{t\in\mathcal{T}}\Theta_{y,t,z}\geq\mu^{RE}_{z}\times\sum_{t\in\mathcal{T}}D_{t,z}, \qquad \forall z\in\mathcal{Z} \tag{6}$$

$$\textstyle\sum_{z\in\mathcal{Z}}\sum_{y\in\mathcal{RE}}\sum_{t\in\mathcal{T}}\Theta_{y,t,z}\geq\sum_{z\in\mathcal{Z}}\sum_{t\in\mathcal{T}}(\mu^{RE}_{z}\times D_{t,z}) \tag{7}$$

$$
\begin{aligned}
&\textstyle\sum_{y\in\mathcal{H}}\Theta_{y,t,z}+\sum_{y\in\mathcal{D}}\Theta_{y,t,z}+\sum_{y\in\mathcal{ND}}\Theta_{y,t,z}\\
&\quad\textstyle+\sum_{y\in\mathcal{O}}(\Theta_{y,t,z}-\Pi_{y,t,z})+\sum_{y\in\mathcal{DR}}(-\Theta_{y,t,z}+\Pi_{y,t,z})\\
&\quad\textstyle-\sum_{y\in\mathcal{HO}}\Pi_{y,t,z}+\sum_{y\in\mathcal{W}}\Theta_{y,t,z}\\
&\quad\textstyle+\sum_{y\in\mathcal{AN}}(\Theta_{y,t,z}+\eta^{heat}_{y,z}(\sigma_{x,t,z}+\nu_{y,t,z}))\\
&\quad\textstyle+\sum_{s\in\mathcal{S}}\Lambda_{s,t,z}-\sum_{l\in\mathcal{L}}(\varphi^{map}_{l,z}\times\Phi_{l,t})\\
&\quad\textstyle-\frac{1}{2}\sum_{l\in\mathcal{L}}(\mid\varphi^{map}_{l,z}\mid\times\beta_{l,t}(\cdot))-\ell_{z,t}=D_{t,z}, && \forall z\in\mathcal{Z},\forall t\in\mathcal{T} \qquad (8)
\end{aligned}
$$

$$-\varphi^{max}_l\le\Phi_{l,t}\le\varphi^{max}_l, \qquad \forall l\in(\mathcal{L}\setminus\mathcal{E}),\forall t\in\mathcal{T} \qquad (9)$$

$$-(\varphi^{max}_l+\triangle\varphi^{max}_l)\le\Phi_{l,t}\le(\varphi^{max}_l+\triangle\varphi^{max}_l), \qquad \forall l\in\mathcal{E},\forall t\in\mathcal{T} \qquad (10)$$

$$\triangle\varphi^{max}_l\le\overline{\triangle\varphi^{max}_l}, \qquad \forall l\in\mathcal{E},\forall t\in\mathcal{T} \qquad (11)$$

$$\Phi_{l,t}=\tfrac{(\varphi^{volt}_l)^2}{\varphi^{ohm}_l}\textstyle\sum_{z\in\mathcal{Z}}(\varphi^{map}_{l,z}\times\vartheta_{z,t}), \qquad \forall l\in\mathcal{L},\forall t\in\mathcal{T} \qquad (12)$$

$$-\varphi^{\theta}_l\le\textstyle\sum_{z\in\mathcal{Z}}(\varphi^{map}_{l,z}\times\vartheta_{z,t})\le\varphi^{\theta}_l, \qquad \forall l\in\mathcal{L},\forall t\in\mathcal{T} \qquad (13)$$

$$\vartheta_{1,t}=0, \qquad \forall t\in\mathcal{T} \qquad (14)$$

$$\beta_{l,t}(\cdot)=\begin{cases}0 & \text{if losses. 0}\\ \varphi^{loss}_l\times\mid\Phi_{l,t}\mid & \text{if losses. 1}\\ \ell_{l,t} & \text{if losses. 2}\end{cases}, \qquad \forall l\in\mathcal{L},\forall t\in\mathcal{T} \qquad (15)$$

$$\Phi_{l,t}=\Phi^+_{l,t}-\Phi^-_{l,t}, \qquad \forall l\in\mathcal{L},\forall t\in\mathcal{T} \qquad (16)$$

$$\mid\Phi_{l,t}\mid=\Phi^+_{l,t}+\Phi^-_{l,t}, \qquad \forall l\in\mathcal{L},\forall t\in\mathcal{T} \qquad (17)$$

$$\Phi^+_{l,t}\le\varphi^{max}_l, \qquad \forall l\in\mathcal{L},\forall t\in\mathcal{T} \qquad (18)$$

$$\Phi^-_{l,t}\le\varphi^{max}_l, \qquad \forall l\in\mathcal{L},\forall t\in\mathcal{T} \qquad (19)$$

$$\ell_{l,t} = \frac{\varphi_l^{ohm}}{(\varphi_l^{volt})^2}\Bigg(\sum_{m\in\mathcal{M}}(S^+_{m,l} \times \S^+_{m,l,t} + S^-_{m,l} \times \S^-_{m,l,t})\Bigg), \qquad \forall l \in \mathcal{L}, \forall t \in \mathcal{T}$$

Where:

$$S^+_{m,l} = \frac{2+4\times\sqrt{2}\times(m-1)}{1+\sqrt{2}\times(2\times M-1)}(\varphi_l^{max} + \overline{\triangle\varphi_l^{max}}) \qquad \forall m \in [1\colon M], l \in \mathcal{L}$$
$$S^-_{m,l} = \frac{2+4\times\sqrt{2}\times(m-1)}{1+\sqrt{2}\times(2\times M-1)}(\varphi_l^{max} + \overline{\triangle\varphi_l^{max}}) \qquad \forall m \in [1\colon M], l \in \mathcal{L} \tag{20}$$

$$\S^+_{m,l,t}, \S^-_{m,l,t} <= \overline{\S_{m,l}} \qquad \forall m \in [1:M], l \in \mathcal{L}, t \in \mathcal{T}$$

Where:

$$\overline{\S_{l,z}} = \begin{cases} \frac{(1+\sqrt{2})}{1+\sqrt{2}\times(2\times M-1)}(\varphi_l^{max} + \overline{\triangle\varphi_l^{max}}) & \text{if m=1} \\ \\ \frac{2\times\sqrt{2}}{1+\sqrt{2}\times(2\times M-1)}(\varphi_l^{max} + \overline{\triangle\varphi_l^{max}}) & \text{if } m > 1 \end{cases}, \tag{21}$$

$$\sum_{m\in[1:M]}(\S^+_{m,l,t}) - \S^+_{0,l,t} = \Phi_{l,t}, \qquad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{22}$$

$$\sum_{m\in[1:M]}(\S^-_{m,l,t}) - \S^-_{0,l,t} = -\Phi_{l,t}, \qquad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{23}$$

$$\S^+_{m,l,t} <= \overline{\S_{m,l}} \times ON^+_{m,l,t}, \qquad \forall m \in [1:M], \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{24}$$

$$\S^-_{m,l,t} <= \overline{\S_{m,l}} \times ON^-_{m,l,t}, \qquad \forall m \in [1:M], \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{25}$$

$$\S^+_{m,l,t} \geq ON^+_{m+1,l,t} \times \overline{\S_{m,l}}, \qquad \forall m \in [1:M], \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{26}$$

$$\S^-_{m,l,t} \geq ON^-_{m+1,l,t} \times \overline{\S_{m,l}}, \qquad \forall m \in [1:M], \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{27}$$

$$\S^+_{0,l,t} \leq \varphi_l^{max} \times (1 - ON^+_{1,l,t}), \qquad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{28}$$

$$\S^-_{0,l,t} \leq \varphi_l^{max} \times (1 - ON^-_{1,l,t}), \qquad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{29}$$

$$\upsilon_{y,t,z} \leq \frac{\overline{\Delta_{y,z}}}{\Omega^{size}_{y,z}} + \Omega_{y,z} - \Delta_{y,z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{30}$$

$$\chi_{y,t,z} \leq \frac{\overline{\Delta_{y,z}}}{\Omega^{size}_{y,z}} + \Omega_{y,z} - \Delta_{y,z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{31}$$

$$\zeta_{y,t,z} \leq \frac{\overline{\Delta_{y,z}}}{\Omega^{size}_{y,z}} + \Omega_{y,z} - \Delta_{y,z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{32}$$

$$\upsilon_{y,t,z} = \upsilon_{y,t-1,z} + \chi_{y,t,z} - \zeta_{y,t,z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{33}$$

$$\Theta_{y,t,z} \geq \rho^{min}_{y,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}, \qquad \forall y \in (\mathcal{UC} \cap \mathcal{H}), \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{34}$$

$$\Theta_{y,t,z} \leq \rho^{max}_{y,t,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}, \qquad \forall y \in (\mathcal{UC} \cap \mathcal{H}), \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{35}$$

$$\begin{aligned}\Theta_{y,t-1,z} - \Theta_{y,t,z} \leq\ & \kappa^{down}_{y,z} \times \Omega^{size}_{y,z} \times (\upsilon_{y,t,z} - \chi_{y,t,z}) \\ & -\rho^{min}_{y,z} \times \Omega^{size}_{y,z} \times \chi_{y,t,z} \\ & + \min(\rho^{max}_{y,t,z}, \max(\rho^{min}_{y,z}, \kappa^{down}_{y,z})) \times \Omega^{size}_{y,z} \times \zeta_{y,t,z}, \qquad \forall y \in (\mathcal{UC} \cap \mathcal{H}), \forall z \in \mathcal{Z}, \forall t \in \mathcal{T}\end{aligned} \tag{36}$$

$$\begin{aligned}\Theta_{y,t,z} - \Theta_{y,t-1,z} \leq\ & \kappa^{up}_{y,z} \times \Omega^{size}_{y,z} \times (\upsilon_{y,t,z} - \chi_{y,t,z}) \\ & + \min(\rho^{max}_{y,t,z}, \max(\rho^{min}_{y,z}, \kappa^{up}_{y,z})) \times \Omega^{size}_{y,z} \times \chi_{y,t,z} \\ & -\rho^{min}_{y,z} \times \Omega^{size}_{y,z} \times \zeta_{y,t,z}, \qquad \forall y \in (\mathcal{UC} \cap \mathcal{H}), \forall z \in \mathcal{Z}, \forall t \in \mathcal{T}\end{aligned} \tag{37}$$

$$\upsilon_{y,t,z} \geq \sum_{\hat{t}=t-\tau^{up}_{y,z}}^{t} \chi_{y,\hat{t},z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{38}$$

$$\left(\frac{\overline{\Delta_{y,z}}}{\Omega^{size}_{y,z}} + \Omega_{y,z} - \Delta_{y,z}\right) - \upsilon_{y,t,z} \geq \sum_{\hat{t}=t-\tau^{up}_{y,z}}^{t} \zeta_{y,\hat{t},z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{39}$$

$$\Theta_{y,t-1,z} - \Theta_{y,t,z} \leq \kappa^{down}_{y,z} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in (\mathcal{H} \setminus \mathcal{UC}), \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{40}$$

$$\Theta_{y,t,z} - \Theta_{y,t-1,z} \leq \kappa^{up}_{y,z} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in (\mathcal{H} \setminus \mathcal{UC}), \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{41}$$

$$\Theta_{y,t,z} \geq \rho^{min}_{y,z} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in (\mathcal{H} \setminus \mathcal{UC}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{42}$$

$$\Theta_{y,t,z} \leq \rho^{max}_{y,t,z} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in (\mathcal{H} \setminus \mathcal{UC}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{43}$$

$$\Theta_{y,t,z} \leq \rho^{max}_{y,t,z} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{D}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{44}$$

$$\Theta_{y,t,z} = \rho^{max}_{y,t,z} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{ND}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{45}$$

$$\Gamma_{y,t,z} = \Gamma_{y,t-1,z} - \left(\frac{\Theta_{y,t,z}}{\eta^{down}_{y,z}}\right) + (\eta^{up}_{y,z} \times \Pi_{y,t,z}) - (\eta^{loss}_{y,z} \times \Gamma_{y,t,z}), \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{46}$$

$$\Gamma_{y,t,z} \leq \frac{(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\mu^{stor}_{y,z}}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{47}$$

$$\Pi_{y,t,z} \leq \frac{(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\eta^{up}_{y,z}}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{48}$$

$$\Pi_{y,t,z} \leq \frac{(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\mu^{stor}_{y,z}} - \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{49}$$

$$\Theta_{y,t,z} \leq \eta^{down}_{y,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{50}$$

$$\Theta_{y,t,z} \leq \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{51}$$

$$\left(\frac{\Theta_{y,t,z}}{\eta^{down}_{y,z}}\right) + (\eta^{up}_{y,z} \times \Pi_{y,t,z}) \leq (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{52}$$

$$\Gamma_{y,t,z} = \Gamma_{y,t-1,z} - \Theta_{y,t,z} + \Pi_{y,t,z}, \qquad \forall y \in \mathcal{DR}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{53}$$

$$\Pi_{y,t,z} \leq \mu^{DMS}_{y,z} \times D_{t,z}, \qquad \forall y \in \mathcal{DR}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{54}$$

$$\sum_{\hat{t}=t+1}^{t+\overline{\tau_{y,z}}} \Theta_{y,\hat{t},z} \geq \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{DR}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{55}$$

$$\Lambda_{s,t,z} \leq (n^{size}_{s} \times D_{t,z}), \qquad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{56}$$

$$\Theta_{y,t,z} = (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in (\mathcal{AN} \setminus \mathcal{UC}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{57}$$

$$\sigma_{x,t,z} + \nu_{y,t,z} \leq \mu^{heat}_{y,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in (\mathcal{AN} \setminus \mathcal{UC}), \forall x \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{58}$$

$$\Theta_{y,t,z} = \rho^{max}_{y,t,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}, \qquad \forall y \in (\mathcal{AN} \cap \mathcal{UC}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{59}$$

$$\sigma_{x,t,z} + \nu_{y,t,z} \leq \mu^{heat}_{y,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}, \qquad \forall y \in (\mathcal{AN} \cap \mathcal{UC}), \forall x \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{60}$$

$$\Gamma_{y,t,z} = \Gamma_{y,t-1,z} - \left(\frac{\Theta_{y,t,z}}{\eta^{down}_{y,z}}\right) + (\eta^{up}_{y,z} \times \Pi_{y,t,z}) - (\eta^{loss}_{y,z} \times \Gamma_{y,t,z}), \qquad \forall y \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{61}$$

$$\Gamma_{y,t,z} \leq \frac{(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\mu^{stor}_{y,z}}, \qquad \forall y \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{62}$$

$$\Pi_{y,t,z} \leq \frac{(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\eta^{up}_{y,z}}, \qquad \forall y \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{63}$$

$$\Pi_{y,t,z} \leq \frac{(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\mu^{stor}_{y,z}} - \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{64}$$

$$\Theta_{y,t,z} \leq \eta^{down}_{y,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{65}$$

$$\Theta_{y,t,z} \leq \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{66}$$

$$\left(\frac{\Theta_{y,t,z}}{\eta^{down}_{y,z}}\right) + (\eta^{up}_{y,z} \times \Pi_{y,t,z}) \leq (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{67}$$

$$\epsilon_{y,t,z} + \sigma_{y,t,z} = \Theta_{y,t,z}, \qquad \forall y \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{68}$$

$$\sum_{y \in \mathcal{HO}} \epsilon_{y,t,z} \leq H_{t,z}, \qquad \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{69}$$

$$\Gamma_{y,t,z} = \Gamma_{y,t-1,z} - \left(\frac{\Theta_{y,t,z}}{\eta^{down}_{y,z}}\right) + \left(\frac{\rho^{max}_{y,t,z} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\mu^{stor}_{y,z}}\right), \qquad \forall y \in \mathcal{W}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{70}$$

$$\Gamma_{y,1,z} = \frac{w^{level}_{y,z} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\mu^{stor}_{y,z}}, \qquad \forall y \in \mathcal{W}, \forall z \in \mathcal{Z} \tag{71}$$

$$\Gamma_{y,t,z} \leq \frac{(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\mu^{stor}_{y,z}}, \qquad \forall y \in \mathcal{W}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{72}$$

$$\Theta_{y,t,z} \leq \eta^{down}_{y,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{W}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{73}$$

$$\Theta_{y,t,z} \leq \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{W}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{74}$$

$$\Theta_{y,t,z} \geq \rho^{min}_{y,z}, \qquad \forall y \in \mathcal{W}, z \in \mathcal{Z} \tag{75}$$

$$\begin{aligned} &F^D \textstyle\sum_{z \in \mathcal{Z}} D_{t,z} + F^{VRE} \sum_{z \in \mathcal{Z}} \sum_{y \in (\mathcal{D} \cup \mathcal{ND})} (\Omega_{y,z} \times \rho^{max}_{y,t,z}) \\ &\quad \leq \textstyle\sum_{y \in \mathcal{G}} \sum_{z \in \mathcal{Z}} f^{+}_{y,z,t}, \qquad \forall t \in \mathcal{T} \end{aligned} \tag{76}$$

$$\begin{aligned} &F^D \textstyle\sum_{z \in \mathcal{Z}} D_{t,z} + F^{VRE} \sum_{z \in \mathcal{Z}} \sum_{y \in (\mathcal{D} \cup \mathcal{ND})} (\Omega_{y,z} \times \rho^{max}_{y,t,z}) \\ &\quad \leq \textstyle\sum_{y \in \mathcal{G}} \sum_{z \in \mathcal{Z}} f^{-}_{y,z,t}, \qquad \forall t \in \mathcal{T} \end{aligned} \tag{77}$$

$$\begin{aligned} &R^{+D} \textstyle\sum_{z \in \mathcal{Z}} D_{t,z} + R^{+VRE} \sum_{z \in \mathcal{Z}} \sum_{y \in (\mathcal{D} \cup \mathcal{ND})} (\Omega_{y,z} \times \rho^{max}_{y,t,z}) + \alpha_t(\cdot) \\ &\quad \leq \textstyle\sum_{y \in \mathcal{G}} \sum_{z \in \mathcal{Z}} r^{+}_{y,z,t} + r^{+,unmet}_{t}, \qquad \forall t \in \mathcal{T} \end{aligned} \tag{78}$$

$$\alpha_t(\cdot) = \begin{cases} \max(\Omega^{size}_{y,z}) & \text{if cont. 1} \\ \max(\max(\Omega^{size}_{y,z}), \max(\varphi^{max}_{l})) & \text{if cont. 2} \\ \max(\max(\Omega^{size}_{y,z}), \max(\varphi^{max}_{l})) & \forall y \mid \Omega_{y,z} \neq 0 \text{ if cont. 3} \\ \max(\max(\Omega^{size}_{y,z}), \max(\varphi^{max}_{l})) & \forall y \mid \upsilon_{y,t,z} \neq 0 \text{ if cont. 4} \end{cases}, \ \forall t \in \mathcal{T} \tag{79}$$

$$\begin{aligned} &R^{-D} \textstyle\sum_{z \in \mathcal{Z}} D_{t,z} + R^{-VRE} \sum_{z \in \mathcal{Z}} \sum_{y \in (\mathcal{D} \cup \mathcal{ND})} (\Omega_{y,z} \times \rho^{max}_{y,t,z}) \\ &\quad \leq \textstyle\sum_{y \in \mathcal{G}} \sum_{z \in \mathcal{Z}} r^{-}_{y,z,t} + r^{-,unmet}_{t}, \qquad \forall t \in \mathcal{T} \end{aligned} \tag{80}$$

$$f^{+}_{y,z,t} \leq \iota^{+}_{y,z}(\Omega^{size}_{y,z} \times \upsilon_{y,t,z}), \qquad \forall y \in (\mathcal{UC} \cap \mathcal{H}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{81}$$

$$f^{-}_{y,z,t} \leq \iota^{-}_{y,z}(\Omega^{size}_{y,z} \times \upsilon_{y,t,z}), \qquad \forall y \in (\mathcal{UC} \cap \mathcal{H}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{82}$$

$$r^{+}_{y,z,t} \leq \gamma^{+}_{y,z}(\Omega^{size}_{y,z} \times \upsilon_{y,t,z}), \qquad \forall y \in (\mathcal{UC} \cap \mathcal{H}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{83}$$

$$r^{-}_{y,z,t} \leq \gamma^{-}_{y,z}(\Omega^{size}_{y,z} \times \upsilon_{y,t,z}), \qquad \forall y \in (\mathcal{UC} \cap \mathcal{H}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{84}$$

$$f^{+}_{y,z,t} \leq \iota^{+}_{y,z} \times \rho^{max}_{y,t,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \notin (\mathcal{UC} \cup \mathcal{ND} \cup \mathcal{DR}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{85}$$

$$f^{-}_{y,z,t} \leq \iota^{-}_{y,z} \times \rho^{max}_{y,t,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \notin (\mathcal{UC} \cup \mathcal{ND} \cup \mathcal{DR}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{86}$$

$$r^{+}_{y,z,t} \leq \gamma^{+}_{y,z} \times \rho^{max}_{y,t,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \notin (\mathcal{UC} \cup \mathcal{ND} \cup \mathcal{DR}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{87}$$

$$r^{-}_{y,z,t} \leq \gamma^{-}_{y,z} \times \rho^{max}_{y,t,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \notin (\mathcal{UC} \cup \mathcal{ND} \cup \mathcal{DR}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{88}$$

$$f^{+}_{y,z,t} = f^{+D}_{y,z,t} + f^{+C}_{y,z,t}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{89}$$

$$f^{-}_{y,z,t} = f^{-D}_{y,z,t} + f^{-C}_{y,z,t}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{90}$$

$$r^{+}_{y,z,t} = r^{+D}_{y,z,t} + r^{+C}_{y,z,t}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{91}$$

$$r^{-}_{y,z,t} = r^{-D}_{y,z,t} + r^{-C}_{y,z,t}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{92}$$

$$\Theta_{y,t,z} - f^{-}_{y,z,t} - r^{-}_{y,z,t} \geq \rho^{min}_{y,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{93}$$

$$\Theta_{y,t,z} + f^{+}_{y,z,t} + r^{+}_{y,z,t} \leq \rho^{max}_{y,t,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{94}$$

$$\Theta_{y,t,z} - f^{-}_{y,z,t} - r^{-}_{y,z,t} \geq \rho^{min}_{y,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{H}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{95}$$

$$\Theta_{y,t,z} + f^{+}_{y,z,t} + r^{+}_{y,z,t} \leq \rho^{max}_{y,t,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{H}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{96}$$

$$\Theta_{y,t,z} - f^{-}_{y,z,t} - r^{-}_{y,z,t} \geq 0, \qquad \forall y \in \mathcal{D}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{97}$$

$$\Theta_{y,t,z} + f^{+}_{y,z,t} + r^{+}_{y,z,t} \leq \rho^{max}_{y,t,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{D}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{98}$$

$$\Pi_{y,t,z} + f^{-C}_{y,z,t} + r^{-C}_{y,z,t} \leq \frac{(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\eta^{up}_{y,z}}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{99}$$

$$\Pi_{y,t,z} + f^{-C}_{y,z,t} + r^{-C}_{y,z,t} \leq \frac{(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\mu^{stor}_{y,z}} - \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{100}$$

$$\Pi_{y,t,z} - f^{+C}_{y,z,t} - r^{+C}_{y,z,t} \geq 0, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{101}$$

$$\Theta_{y,t,z} + f^{+D}_{y,z,t} + r^{+D}_{y,z,t} \leq \eta^{down}_{y,z} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{102}$$

$$\Theta_{y,t,z} + f^{+D}_{y,z,t} + r^{+D}_{y,z,t} \leq \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{103}$$

$$\Theta_{y,t,z} - f^{-D}_{y,z,t} + r^{-D}_{y,z,t} \geq 0, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{104}$$

$$\left(\frac{\Theta_{y,t,z} + f^{+D}_{y,z,t} + r^{+D}_{y,z,t}}{\eta^{down}_{y,z}}\right) + \left(\eta^{up}_{y,z}(\Pi_{y,t,z} + f^{-C}_{y,z,t} + r^{-C}_{y,z,t})\right) \leq (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{105}$$

$$\Theta_{y,t,z} + f^{+}_{y,z,t} + r^{+}_{y,z,t} \leq \eta^{down}_{y,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{W}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{106}$$

$$\Theta_{y,t,z} + f^{+}_{y,z,t} + r^{+}_{y,z,t} \leq \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{W}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{107}$$

$$\Theta_{y,t,z} - f^{-}_{y,z,t} - r^{-}_{y,z,t} \geq 0, \qquad \forall y \in \mathcal{W}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{108}$$

$$f^{+}_{y,z,t} \leq \iota^{+}_{y,z}(\eta^{heat}_{y,z} \times \mu^{heat}_{y,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}), \qquad \forall y \in (\mathcal{UC} \cap \mathcal{AN}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{109}$$

$$f^{-}_{y,z,t} \leq \iota^{-}_{y,z}(\eta^{heat}_{y,z} \times \mu^{heat}_{y,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}), \qquad \forall y \in (\mathcal{UC} \cap \mathcal{AN}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{110}$$

$$r^{+}_{y,z,t} \leq \gamma^{+}_{y,z}(\eta^{heat}_{y,z} \times \mu^{heat}_{y,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}), \qquad \forall y \in (\mathcal{UC} \cap \mathcal{AN}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{111}$$

$$r^{-}_{y,z,t} \leq \gamma^{-}_{y,z}(\eta^{heat}_{y,z} \times \mu^{heat}_{y,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}), \qquad \forall y \in (\mathcal{UC} \cap \mathcal{AN}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{112}$$

$$\sigma_{x,t,z} + \nu_{y,t,z} - \frac{(f^{-}_{y,z,t} + r^{-}_{y,z,t})}{\eta^{heat}_{y,z}} \geq 0, \qquad \forall y \in (\mathcal{UC} \cap \mathcal{AN}), \forall x \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{113}$$

$$\sigma_{x,t,z} + \nu_{y,t,z} + \frac{(f^{+}_{y,z,t} + r^{+}_{y,z,t})}{\eta^{heat}_{y,z}} \leq \mu^{heat}_{y,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}, \qquad \forall y \in (\mathcal{UC} \cap \mathcal{AN}), \forall x \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{114}$$

$$\Pi_{y,t,z} + f^{-}_{y,z,t} + r^{-}_{y,z,t} \leq \frac{(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\eta^{up}_{y,z}}, \qquad \forall y \in \mathcal{OH}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{115}$$

$$\Pi_{y,t,z} + f^{-}_{y,z,t} + r^{-}_{y,z,t} \leq \frac{(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\mu^{stor}_{y,z}} - \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{OH}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{116}$$

$$\Pi_{y,t,z} - f^{+}_{y,z,t} - r^{+}_{y,z,t} \geq 0, \qquad \forall y \in \mathcal{OH}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{117}$$

$$\ell_{z,t} = \varphi_z^{Net} \sum_{m=1}^{M} \left(S_{m,z}^{+} \times \S_{m,z,t}^{+} + S_{m,z}^{+} \times \S_{m,z,t}^{-} \right) + \varphi_z^{W} \times W_{z,t} + \varphi_z^{I} \times I_{z,t} + \varphi_z^{Int}, \qquad \forall z \in \mathcal{V}, \forall t \in \mathcal{T}$$

Where:

$$S_{m,z}^{+} = \frac{2+4\times\sqrt{2}\times(m-1)}{1+\sqrt{2}\times(2\times M-1)}(\lambda_z^{I} + \overline{\triangle\lambda_z^{I}}) \qquad \forall m \in [1\colon M], z \in \mathcal{V}$$

$$S_{m,z}^{-} = \frac{2+4\times\sqrt{2}\times(m-1)}{1+\sqrt{2}\times(2\times M-1)}(\lambda_z^{W} + \overline{\triangle\lambda_z^{W}}) \qquad \forall m \in [1\colon M], z \in \mathcal{V}$$

$$W_{z,t} = \sum_{d\in\mathcal{V}} \varphi_{z,d}^{down} \times \left(D_{z,t} + \sum_{y\in\mathcal{O},\mathcal{HO}}(\Pi_{y,t,z}) + \sum_{y\in\mathcal{DR}}(\Theta_{y,t,z}) \right), \qquad \forall z \in \mathcal{V}, t \in \mathcal{T}$$

$$I_{z,t} = \sum_{y\notin\mathcal{DR}}(\Theta_{y,t,z}) \qquad \forall z \in \mathcal{V}, t \in \mathcal{T} \tag{118}$$

$$\S_{m,z,t}^{+}, \leq \overline{\S_{m,z}^{+}}, \qquad \forall m \in [1\colon M], z \in \mathcal{V}, t \in \mathcal{T}$$

Where:

$$\overline{\S_{m,z}^{+}} = \begin{cases} \frac{(1+\sqrt{2})}{1+\sqrt{2}\times(2\times M-1)}(\lambda_z^{I} + \overline{\triangle\lambda_z^{I}}) & \text{if m=1} \\ \\ \frac{2\times\sqrt{2}}{1+\sqrt{2}\times(2\times M-1)}(\lambda_z^{I} + \overline{\triangle\lambda_z^{I}}) & \text{if } m > 1 \end{cases} \tag{119}$$

$$\S_{m,z,t}^{-}, \leq \overline{\S_{m,z}^{-}}, \qquad \forall m \in [1\colon M], z \in \mathcal{V}, t \in \mathcal{T}$$

Where:

$$\overline{\S_{m,z}^{-}} = \begin{cases} \frac{(1+\sqrt{2})}{1+\sqrt{2}\times(2\times M-1)}(\lambda_z^{W} + \overline{\triangle\lambda_z^{W}}) & \text{if m=1} \\ \\ \frac{2\times\sqrt{2}}{1+\sqrt{2}\times(2\times M-1)}(\lambda_z^{W} + \overline{\triangle\lambda_z^{W}}) & \text{if } m > 1 \end{cases}, \tag{120}$$

$$\sum_{m=1}^{M} \left(\S_{m,z,t}^{+} \right) - \S_{0,z,t}^{+} = (W_{z,t} - I_{z,t}) \qquad \forall z \in \mathcal{V}, \forall t \in \mathcal{T} \tag{121}$$

$$\sum_{m=1}^{M} \left(\S_{m,z,t}^{-} \right) - \S_{0,z,t}^{-} = -(W_{z,t} - I_{z,t}) \qquad \forall z \in \mathcal{V}, \forall t \in \mathcal{T} \tag{122}$$

$$\S_{m,z,t}^{+} \leq S_{m,z,t}^{+} \times ON_{m,z,t}^{+} \qquad \forall m \in [1\colon M], z \in \mathcal{V}, t \in \mathcal{T} \tag{123}$$

$$\S_{m,z,t}^{-} \leq S_{m,z,t}^{-} \times ON_{m,z,t}^{-} \qquad \forall m \in [1\colon M], z \in \mathcal{V}, t \in \mathcal{T} \tag{124}$$

$$\S_{m,z,t}^{+} \geq ON_{m+1,z,t}^{+} \times \overline{\S_{m,z}^{+}} \qquad \forall m \in [1\colon (M-1)], z \in \mathcal{V}, t \in \mathcal{T} \tag{125}$$

$$\S_{m,z,t}^{-} \geq ON_{m+1,z,t}^{-} \times \overline{\S_{m,z}^{-}} \qquad \forall m \in [1\colon (M-1)], z \in \mathcal{V}, t \in \mathcal{T} \tag{126}$$

$$\S^{+}_{0,z,t} \leq (\lambda^W_z + \overline{\triangle\lambda^W_z}) \times (1 - ON^{+}_{1,z,t}), \qquad \forall z \in \mathcal{V}, \forall t \in \mathcal{T} \tag{127}$$

$$\S^{-}_{0,z,t} \leq (\lambda^I_z + \overline{\triangle\lambda^I_z}) \times (1 - ON^{-}_{1,z,t}), \qquad \forall z \in \mathcal{V}, \forall t \in \mathcal{T} \tag{128}$$

$$\lambda^W_z + \triangle\lambda^W_z + \phi^W_{z,t} \geq W_{z,t}, \qquad \forall z \in \mathcal{V}, t \in \mathcal{PW} \tag{129}$$

$$\lambda^I_z + \triangle\lambda^I_z + \phi^I_{z,t} \geq I_{z,t}, \qquad \forall z \in \mathcal{V}, t \in \mathcal{PI} \tag{130}$$

$$\triangle\lambda^W_z \leq \overline{\triangle\lambda^W_z}, \qquad \forall z \in \mathcal{V} \tag{131}$$

$$\triangle\lambda^I_z \leq \overline{\triangle\lambda^I_z}, \qquad \forall z \in \mathcal{V} \tag{132}$$

$$\phi^W_{z,t} = N^{W,sqrt}_z \times \sum_{m=1}^{M} \left(S^{\phi,W}_{m,z} \times \S^{\phi,W}_{m,z,t} \right) + N^{W,linear}_z \times DER^W_{z,t} \qquad \forall z \in \mathcal{Z}, t \in \mathcal{PW}$$

Where:

$$DER^W_{z,t} = \sum_{d \in \mathcal{V}} \varphi^{down}_{z,d} \times \left(\sum_{y \notin \mathcal{ND},\mathcal{DR}} (\Theta_{y,t,z}) + \sum_{y \in DR} (\Pi_{y,t,z}) + \sum_{s \in S} (\Lambda_{s,t,z}) \right), \qquad \forall z \in \mathcal{Z}, t \in \mathcal{PW} \tag{133}$$

$$\sum_{m=1}^{M} \S^{\phi,W}_{m,z} = DER^W_{z,t} \qquad \forall z \in \mathcal{Z}, t \in \mathcal{PW} \tag{134}$$

$$\S^{\phi,W}_{m,z} \leq \overline{\S^{\phi,W}_{m,z}} \qquad \forall m \in [1:M], z \in \mathcal{Z}$$

Where:

$$\overline{\S^{\phi,W}_{m,z}} = \frac{6 \times \overline{\phi^W_z}}{M \times (M+1) \times (2 \times M + 1)} \times m^2 \qquad \forall m \in [1:M], z \in \mathcal{V} \tag{135}$$

$$\phi^I_{z,t} = 0, \qquad \forall z \in \mathcal{Z}, t \in \mathcal{T} \tag{136}$$

$$\Omega_{y,z},\ \Delta_{y,z} \geq 0, \qquad \forall y \notin \mathcal{UC}, \forall z \in \mathcal{Z} \tag{137}$$

$$\Omega_{y,z},\ \Delta_{y,z} \geq 0 \in \mathbb{Z}_+, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z} \tag{138}$$

$$\ell_{l,t},\ \Phi^{+}_{l,t},\ \Phi^{-}_{l,t} \geq 0, \qquad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{139}$$

$$\S^{+}_{m,l,t},\ \S^{-}_{m,l,t} \geq 0, \qquad \forall m \in \mathcal{M}, \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{140}$$

$$\triangle\Phi^{m,+,ON}_{l,t},\ \ \triangle\,\Phi^{m,-,ON}_{l,t} \in \{0,1\}, \qquad \forall m \in \mathcal{M}, \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{141}$$

$$\triangle\varphi^{max}_l \geq 0, \qquad \forall l \in \mathcal{E} \tag{142}$$

$$\triangle\varphi_l^{max} = 0, \qquad \forall l \notin \mathcal{E} \tag{143}$$

$$\Theta_{y,t,z} \geq 0, \qquad \forall y \in \mathcal{G}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{144}$$

$$\Pi_{y,t,z} \geq 0, \qquad \forall y \in (\mathcal{O} \cup \mathcal{HO} \cup \mathcal{DR}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{145}$$

$$\Gamma_{y,t,z} \geq 0, \qquad \forall y \in (\mathcal{O} \cup \mathcal{HO} \cup \mathcal{DR} \cup \mathcal{W}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{146}$$

$$\Pi_{y,t,z} = 0, \qquad \forall y \notin (\mathcal{O} \cup \mathcal{HO} \cup \mathcal{DR}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{147}$$

$$\Gamma_{y,t,z} = 0, \qquad \forall y \notin (\mathcal{O} \cup \mathcal{HO} \cup \mathcal{DR} \cup \mathcal{W}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{148}$$

$$\Lambda_{s,t,z} \geq 0, \qquad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{149}$$

$$\upsilon_{y,t,z},\ \chi_{y,t,z},\ \zeta_{y,t,z} = 0, \qquad \forall y \notin \mathcal{UC}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{150}$$

$$\upsilon_{y,t,z},\ \chi_{y,t,z},\ \zeta_{y,t,z} \geq 0 \in \mathbb{Z}_+, \qquad \forall y \in \mathcal{UC}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{151}$$

$$\epsilon_{y,t,z},\ \sigma_{y,t,z} = 0, \qquad \forall y \notin \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{152}$$

$$\epsilon_{y,t,z},\ \sigma_{y,t,z} \geq 0, \qquad \forall y \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{153}$$

$$\nu_{y,t,z} = 0, \qquad \forall y \notin \mathcal{AN}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{154}$$

$$\nu_{y,t,z} \geq 0, \qquad \forall y \in \mathcal{AN}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{155}$$

$$r^+_{y,z,t}, r^-_{y,z,t}, f^+_{y,z,t}, f^-_{y,z,t} \geq 0, \qquad \forall y \notin (\mathcal{ND} \cup \mathcal{DR}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{156}$$

$$r^+_{y,z,t}, r^-_{y,z,t}, f^+_{y,z,t}, f^-_{y,z,t} = 0, \qquad \forall y \in (\mathcal{ND} \cup \mathcal{DR}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{157}$$

$$r^{+C}_{y,z,t}, r^{-C}_{y,z,t}, f^{+C}_{y,z,t}, f^{-C}_{y,z,t}, r^{+D}_{y,z,t}, r^{-D}_{y,z,t}, f^{+D}_{y,z,t}, f^{-D}_{y,z,t} \geq 0, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{158}$$

$$r^{+C}_{y,z,t}, r^{-C}_{y,z,t}, f^{+C}_{y,z,t}, f^{-C}_{y,z,t}, r^{+D}_{y,z,t}, r^{-D}_{y,z,t}, f^{+D}_{y,z,t}, f^{-D}_{y,z,t} = 0, \qquad \forall y \notin \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{159}$$

$$r_t^{+,unmet}, r_t^{-,unmet} \geq 0, \qquad \forall t \in \mathcal{T} \tag{160}$$

$$\ell_{z,t} \geq 0 \qquad \forall z \in \mathcal{V}, t \in \mathcal{T} \tag{161}$$

$$\S^{+}_{m,z,t}, \S^{-}_{m,z,t} \geq 0, \qquad \forall m \in [1:M], z \in \mathcal{V}, t \in \mathcal{T} \tag{162}$$

$$\triangle\lambda^{W}_{z}, \triangle\lambda^{I}_{z} \geq 0, \qquad \forall z \in \mathcal{V} \tag{163}$$

$$\phi^{W}_{z,t}, \phi^{I}_{z,t} \geq 0, \qquad \forall z \in \mathcal{V}, t \in \mathcal{T} \tag{164}$$

$$\S^{\phi,W}_{m,z,t} \geq 0, \qquad \forall m \in [1:M], z \in z\mathcal{V}, t \in \mathcal{T} \tag{165}$$

5 Description of the Model

This section describes the different indices and sets used in the formulation; and then it describes the different general-form equations that make up the model under various possible configurations.

5.1 Indices and Sets

Five indices are used in this model: y, x represent technologies $\in \mathcal{G}$; t, e represent hours $\in \mathcal{T}$; z represents a zone/node $\in \mathcal{Z}$; l represents a transmission line $\in \mathcal{L}$; and s represents a consumers' segment $\in \mathcal{S}$.

$\mathcal{G}$ denotes the set of resources that can be built/deployed, containing all the different thermal technologies (nuclear, coal, combined cycle gas turbines (CCGTs), open cycle gas turbines (OCGTs)), different variable renewable energy technologies (wind, solar photovoltaic, solar thermal), different storage technologies (pumped hydro, batteries, compressed air (CAES), heat storage), various hydroelectric generators (run-of-river and reservoir hydro), as well as demand side mechanisms like shiftable demand. In addition, $\mathcal{H} \subset \mathcal{G}$ denotes the subset of thermal generation resources (coal, nuclear, CCGT, OCGT, etc.); $\mathcal{D} \subset \mathcal{G}$, denotes the subset "dispatchable" (e.g., curtailable) variable renewable resources (solar and wind); $\mathcal{ND} \subset \mathcal{G}$, denotes the subset of "non-dispatchable" (e.g., non-curtailable) renewable capacity (such as rooftop solar PV); $\mathcal{O} \subset \mathcal{G}$, denotes the subset of electrochemical and mechanical storage technologies; $\mathcal{DR} \subset \mathcal{G}$, denotes the subset of demand response technologies or shiftable load; $\mathcal{AN} \subset \mathcal{G}$, denotes the subset of advance nuclear technologies (Nuclear Air Combined Cycle (NACC)); $\mathcal{HO} \subset \mathcal{G}$, denotes the subset of heat storage technologies (electrically-heated thermal storage); $\mathcal{W} \subset \mathcal{G}$, denotes the subset of hydro reservoir resources; $\mathcal{UC} \subset \mathcal{G}$, denotes the subset of thermal and advance nuclear technologies for which unit commitment constraints are considered (this set may be empty if unit commitment decisions are not considered for any resources); and $\mathcal{RE} \subset \mathcal{G}$, denotes the subset of renewable energy resources that qualify to fulfill a minimum renewable energy generation requirement or mandate (e.g., a renewable portfolio standard requirement), if any is modeled.

$\mathcal{T}$ denotes the set of hours modeled in the simulation (e.g., 8760 hours in a full year, or some reduced set of representative hours).

$\mathcal{Z}$ denotes the number of nodes/zones simulated in the analysis. This number could range from 1, when the network is represented as a single node, to very high numbers when trying to describe the real topology of the network at a nodal resolution. In addition, $\mathcal{R} \subset \mathcal{Z}$ denotes the subset of nodes at the transmission level; and $\mathcal{V} \subset \mathcal{Z}$ denotes the subset of nodes at the distribution level.

$\mathcal{S}$ denotes multiple blocks or segments of curtailable demand, in order to represent price responsive demand curtailment or a piecewise approximation of increasing willingness to pay for electricity at different price levels.

$\mathcal{L}$ denotes the number of transmission lines simulated in the analysis. This number could range from 0, when the network is represented as a single node, to very high numbers when trying to describe the real topology of the network. In addition, $\mathcal{E} \subset \mathcal{L}$ denotes the subset of transmission lines eligible for reinforcements or construction.

5.2 Decision Variables

The GenX model can operate in either a "greenfield" or "brownfield" mode for each technology independently. That is, it can solve starting from an existing brownfield capacity mix, $\overline{\Delta_{y,z}}$, while making decisions about building new capacity, $\Omega_{y,z}$, and retiring existing capacity, $\Delta_{y,z}$. Otherwise, GenX solves a greenfield expansion problem assuming no existing capacity, making decisions about new capacity, $\Omega_{y,z}$, from scratch. Maximum installed capacity can be specified for each resource type and each zone (see Eq. 2) to reflect constraints on the siting resources. When operating in brownfield mode, constraint Eq. 3 ensures that total retired capacity does not exceed the initial installed capacity.

Capacity additions for all units of specific technology y in zone z belonging to the set $\mathcal{UC}$ are treated as integer decisions to add a fixed and indivisible increment of capacity $\Omega_{y,z}^{size}$ (e.g., 200 MW or 600 MW). For all other technologies $\notin \mathcal{UC}$, the investment decision variables take continuous positive values i.e., the model could decide to install, for example, 10.5 MW of capacity for solar, wind or thermal generators that do not belong to the set $\mathcal{UC}$. When clustered unit commitment is activated for technology y, then the investment decision variable can take only positive integer variables that represent the number of plants in cluster y i.e., the model could decide to install, for example, 5 units with a total generation capacity equal to the number of plants times the plants size of the cluster $\Omega_{y,z}^{size}$.

For any given technology y in zone z, it is possible to generate/inject, $\Theta_{y,t,z}$, of energy at each hour t. The maximum amount of generation/injection will be limited by the net capacity installed—equal to $\Omega_{y,z}^{size}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})$, or total existing capacity plus installed additions less retirements—and a set of different operational constraints that are applied depending on the type (subset) of the specific technology y. At the same time, for energy storage devices belonging to $\mathcal{O}$, energy can be stored/withdraw, $\Pi_{y,t,z}$, at each hour t. The net change in energy stored, $\Gamma_{y,t,z}$, at time t will depend on the amount of energy discharged, $\Theta_{y,t,z}$, the amount of energy charged, $\Pi_{y,t,z}$, and the storage energy level at time $t-1$. For non-storage technologies the variables for charge and storage level are set to zero or eliminated from the problem.

For every hour, the system must resolve how much energy to inject/withdraw in each zone z given the hourly demand $D_{t,z}$ and the variable costs of the different electricity resource options. Consumers present different preferences about their willingness to pay for electricity at different prices; i.e., if the hourly marginal cost of electricity supply goes above a consumer's marginal value of consumption, that consumer is better off not consuming and will thus curtail demand. For this purpose, the model can decide how much demand not to serve $\Lambda_{s,t,z}$ at time t in zone z with different costs for each segment of demand s. Each segment is represented by a per unit portion n_s^{size} of the hourly demand in zone z that is willing to pay no more to consume energy than the specified price n_s^{slope} per unit energy.

When more than one zone is modeled, energy can flow, $\Phi_{l,t}$, through the line l at time t connecting zones according to the network map, $\varphi^{map}_{l,z}$. Different lines present different characteristics like maximum capacity, φ^{max}_{l}, transmission voltage, φ^{volt}_{l}, and resistivity, φ^{ohm}_{l}. The model can be configured for one of two approximations of network power flows: a DC power flow approximation, or a transport model. Under a DC power flow configuration, the power flow, $\Phi_{l,t}$, at any given time t will depend on the line parameters and the difference between zone/bus angle, $\vartheta_{z,t}$, that a particular line connects. The power flow variable, $\Phi_{l,t}$, for any line connecting nodes i and j can be either positive or negative, representing flows from i to j (positive values) or from j to i (negative values) at any given time. If the model is configured to allow network expansion, then $\forall\ l \in \mathcal{E}$, maximum transmission capacity, φ^{max}_{l} can be in increased by investing in line reinforcements, $\triangle\varphi^{max}_{l}$, at a cost of π^{TCAP}_{l}. Reinforcements are limited by the maximum reinforcement capacity, $\overline{\triangle\varphi^{max}_{l}}$ of each line and the current version of model considers reinforcements to be continuous variables. For lines that are not eligible for reinforcements ($\forall\ l \notin \mathcal{E}$), $\triangle\varphi^{max}_{l}$ are set to zero or eliminated from the problem. Additionally, different approximations for transmission losses, $\ell_{l,t}$, can be used. Depending on the approximation used, different auxiliary variables—$\Phi^{+}_{l,t}, \Phi^{-}_{l,t}, \S^{+}_{m,l,t}, \S^{-}_{m,l,t}, ON^{+}_{m,l,t}, ON^{-}_{m,l,t}$—are be used by the model to ensure that no "phantom losses" exist i.e., fake losses created to avoid cycling by thermal units and save on startup costs [38].

For generators $y \in \mathcal{UC}$, it is possible to combine similar generating units into clusters of similar or identical units using the integer clustering method developed by [17–19]. This replaces the large set of binary commitment decisions and associated constraints (which scale with the number of individual generating units $y \in \mathcal{UC}$), with a smaller set of integer commitment states and constraints (which instead scale with the number of clusters y of alike units). The distinction between binary and integer variables is important: in traditional binary unit commitment, each unit is either on or off. With clustering, the integer commitment state varies from zero to the number of units in the cluster, $(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})$, with the commitment variable representing the number of individual units in the cluster committed. In this way, clustering still captures integer commitment decisions and associated relations at the individual plant level, subject to the simplifying assumption that all clustered units have identical parameters (e.g., capacity size, ramp rates, heat rate) and that all committed units in a given time step t are operating at the same power output per unit (see [17–19, 23] for detailed discussion of this method). All of the other generator variables—such as power output level—and constraints are then aggregated for the entire cluster. For each cluster y in zone z there will be a commitment state variable $\upsilon_{y,t,z}$ at every hour t that indicates for that cluster the number of generators that are committed. Then, in order to change this number, startup events $\chi_{y,t,z}$ or shutdown events $\zeta_{y,t,z}$ will be needed. For technologies $y \notin \mathcal{UC}$, unit commitment decisions do not apply, and all associated decision variables are eliminated from the problem.

GenX can also model two classes of operating reserves required by system operators to maintain supply-demand balance at all nodes within tolerances in the case of unexpected generator or transmission line failures or errors in demand or renewable energy forecasts within the hourly time step considered in the model. In this way, GenX allows a robust treatment of important stochastic processes not explicitly modeled. Primary reserve (or frequency regulation) requirements are symmetric (e.g., equal in both upward and downward directions) and are determined as a linear function of the hourly demand and the hourly generation from VRE resources, F^{D} and F^{VRE}, respectively. Different technologies can contribute some portion of their capacity to primary reserves up to a maximum, $\iota^{+}_{y,z}$ and $\iota^{-}_{y,z}$, for regulation up and down respectively. Primary reserves provided by the different resources at each hour, $f^{+}_{y,z,t}$ and $f^{-}_{y,z,t}$, must at least fulfill the total primary reserve requirements at each hour. For storage technologies either charging or discharging processes can provide primary reserves. The total amount of primary reserves provided by storage technologies for reserves up, $f^{+C}_{y,z,t} + f^{+D}_{y,z,t}$, and down, $f^{-C}_{y,z,t} + f^{-D}_{y,z,t}$, equals the sum provided by each process, charging and discharging. Primary reserve requirements are considered a hard constraint, and primary reserve shortfalls are not permitted. In addition, secondary reserves (e.g., spinning reserves or balancing energy) requirements can be specified asymmetrically (with different requirements for upwards and downward directions) again, as a linear function of demand, R^{+D} and R^{-D}, and VRE generation, R^{+VRE} and R^{-VRE}, for each hour. Different technologies can commit capacity to secondary reserves up to a maximum, $\gamma^{+}_{y,z}$ and $\gamma^{-}_{y,z}$, for regulation up and down respectively. In addition, secondary reserve shortfalls, $r^{+,unmet}_{t}$ and $r^{-,unmet}_{t}$, are permitted with a penalty for shortfalls imposed at the cost π^{unmet} per MW-hr. Secondary reserves provided by the different resources at each hour, $r^{+}_{y,z,t}$ and $r^{-}_{y,z,t}$, minus unmet secondary reserves, $r^{+,unmet}_{t}$ and $r^{-,unmet}_{t}$, must fulfill the total secondary reserve requirements at each hour. Again, the total amount of reserves provided by storage for secondary reserves provided by storage technologies for reserves up, $r^{+C}_{y,z,t} + r^{+D}_{y,z,t}$, and down, $r^{-C}_{y,z,t} + r^{-D}_{y,z,t}$, equals the sum provided by each process, charging and discharging.

The current version of GenX also allows consideration of two technologies linking the heating and electricity sectors: electrically-heated thermal energy storage and Nuclear Air-Brayton Combined Cycle (NACC) generators that feature a thermal topping cycle to increase production above the normal output levels (see [9, 41]). Electrically-heated thermal storage technologies may discharge stored energy (heat), $\Theta_{y,t,z}$, for one of two uses. First, if heat-electricity market interactions are allowed, then a fraction of the heat (that ranges from 0% to 100%) could go to the heat market, $\epsilon_{y,t,z}$, to be sold at the heat market price, π^{HEAT}_{z}, of that zone in order to satisfy some fraction of the heat demand, $H_{t,z}$. Second, if NACC technologies are present in the mix, then a fraction of the heat (that ranges from 0% to 100%) could be directed towards electricity generation, $\sigma_{y,t,z}$, via the NACC unit's topping cycle, which increases the plant's electrical output above the normal cycle with a peak efficiency, $\eta^{heat}_{y,z}$, for a maximum fraction,$\mu^{heat}_{y,z}$, of the NACC installed capacity. Additionally, heat from combustion of natural gas, $\nu_{y,t,z}$, can be bought at the market price, π^{HEAT}_{z}, and used for peak generation at the NACC units.

Finally, GenX can model distribution network losses and constraints on aggregate peak injections and withdrawals within each distribution zone. In this case, $\ell_{z,t}$ equals the losses from power flows due to aggregate withdrawals and injections within each zone z at each time t. Accurately representing distribution network losses entails a segmentwise linear approximation of a quadratic function of net withdrawals in each zone, and to do so, GenX employs a set of auxiliary variables—$S^{+}_{m,z,t}, \S^{-}_{m,z,t}, ON^{+}_{m,z,t}, ON^{-}_{m,z,t}$. Aggregate withdrawals and injections are constrained in each zone, and distribution network capacity can expanded to accommodate additional withdrawals ($\triangle\lambda^{W}_{z}$) or injections ($\triangle\lambda^{I}_{z}$). In addition, GenX can model the additional aggregate peak withdrawals that can be accommodated via optimal dispatch of distributed energy resources (DERs) and demand response. The network margin gained via dispatch of DERs is denoted by the variables $\phi^{W}_{z,t}$ and $\phi^{I}_{z,t}$ and the current implementation of the model employs auxiliary variables $\S^{\phi,W}_{m,z,t}$ to represent a segmentwise approximation of a square-root function in calculation of this additional network margin gained.

5.3 Objective Function

The Objective Function in this model (see Eq. 1) minimizes over 7 components:

$$
\begin{aligned}
\min\Bigg\{ & \sum_{z\in\mathcal{Z}}\sum_{y\in\mathcal{G}}\Big((\pi^{INVEST}_{y,z}\times\Omega^{size}_{y,z}\times\Omega_{y,z})+(\pi^{FOM}_{y,z}\times\Omega^{size}_{y,z}\times(\overline{\Delta_{y,z}}+\Omega_{y,z}-\Delta_{y,z}))\Big)\\
&+\sum_{z\in\mathcal{Z}}\sum_{y\in\mathcal{G}}\sum_{t\in\mathcal{T}}\Big(((\pi^{VOM}_{y,z}+\pi^{FUEL}_{y,z})\times\Theta_{y,t,z})+(\pi^{VOM}_{y,z}\times\Pi_{y,t,z})+(\pi^{HEAT}_{z}\times\nu_{y,t,z})\Big)\\
&+\sum_{z\in\mathcal{Z}}\sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}}\Big(n^{slope}_{s}\times\Lambda_{s,t,z}\Big)\\
&+\sum_{z\in\mathcal{Z}}\sum_{y\in\mathcal{G}}\sum_{t\in\mathcal{T}}\Big(\pi^{START}_{y,z}\times\chi_{y,t}\Big)\\
&-\sum_{z\in\mathcal{Z}}\sum_{y\in\mathcal{G}}\sum_{t\in\mathcal{T}}\Big(\pi^{HEAT}_{z}\times\epsilon_{y,t,z}\Big)\\
&+\sum_{t\in\mathcal{T}}\Big(\pi^{unmet}(r^{+,unmet}_{t}+r^{-,unmet}_{t})\Big)\\
&+\sum_{l\in\mathcal{L}}\Big(\pi^{TCAP}_{l}\times\triangle\varphi^{max}_{l}\Big)\\
&+\sum_{z\in\mathcal{V}}\Big(\pi^{DCAP}_{z}\times(\triangle\lambda^{W}_{z}+\triangle\lambda^{I}_{z})\Big)\Bigg\}
\end{aligned}
\tag{1}
$$

The first component represents the minimization over the fixed cost across all zones and technologies, which reflects the sum of the annualized capital cost, $\pi^{INVEST}_{y,z}$, times the total new capacity added (if any), plus the fixed O&M cost, $\pi^{FOM}_{y,z}$, times the net installed capacity (e.g., existing capacity less retirements plus additions). The second term corresponds to operational cost minimization across all zones, technologies, and hours, representing the sum of fuel cost, $pi^{FUEL}_{y,z}$, plus variable O&M cost, $\pi^{VOM}_{y,z}$, times the hourly energy injection, $\Theta_{y,t,z}$, the variable cost, $\pi^{VOM}_{y,z}$, times the energy withdrawn or stored by storage resources, $\Pi_{y,t,z}$, and (if applicable) the heat cost, π^{HEAT}_{z}, times the amount of heat from natural gas, $\nu_{y,t,z}$, utilized. The third term minimizes the cost of unserved demand across all different consumer segments s, equal to the marginal value of consumption (or cost of non-served energy) times the amount of non-served energy, $\Lambda_{s,t,z}$, for each segment on each zone during each hour. The fourth term corresponds to the startup costs incurred by technologies to which unit commitment decisions apply (e.g., $\forall\ y \in \mathcal{UC}$), equal to the cost of start-up $\pi^{START}_{y,z}$, times the number of startup events, $\chi_{y,t}$, for the cluster of units in each zone and time step. The fourth term corresponds to the maximization of revenues earned from selling thermal output to the heat market (if applicable). This is, the sum across all zones, technologies, and hours of the heat sales, $\epsilon_{y,t,z}$, times the zonal heat price, π^{HEAT}_{z}. The fifth term correspond to cost for the system of any unmet secondary reserve requirements equal to the sum across all hours of the cost of unmet reserves, π^{unmet}, times the sum of unmet secondary up and down reserves, $r^{+,unmet}_{t}+r^{-,unmet}_{t}$. Finally, the last two terms correspond to the network reinforcement or construction costs, for both transmission lines and distribution zones. Transmission reinforcement costs are equal to the sum across all lines of the product between the transmission reinforcement/construction cost, π^{TCAP}_{l}, times the additional transmission capacity variable, $\triangle\varphi_l^{max}$. Distribution network reinforcement costs are likewise equal to the sum across all distribution zones ($z \in \mathcal{V}$, if any) of the product between the distribution reinforcement/construction cost, π_l^{DCAP}, times the sum of additional distribution withdrawal ($\lambda_z^{\Delta,W}$) and injection ($\lambda_z^{\Delta,I}$) capacity reinforcements added.

In summary, Eq. 1 can be understood as the minimization of costs associated with eight sets of different decisions: (1) where and how to invest on capacity, (2) how to dispatch that capacity, (3) which consumer segments to serve or curtail, (4) how to cycle and commit thermal units subject to unit commitment decisions, (5) how much heat produce and sell to heat markets, (6) how to provide operating reserves (7), and where and how to invest in additional transmission network capacity to increase power transfer capacity between zones, and (6) where and how to investment in reinforcements of distribution network capacity to accommodate increases in aggregate peak withdrawals or injections within each distribution network. Note however that each of these components are considered jointly and the optimization is performed over the whole problem at once as a monolithic co-optimization problem.

5.4 Accounting for CO_2 Emissions Limits

For every technology y, the parameter $\epsilon_{y,z}^{CO_2}$ reflects the specific carbon emissions intensity in tCO_2/MWh associated with operation. A constraint or cap on total CO_2 emissions intensity in tCO_2/MWh, ϵ_z^{max}, can be established for each zone. Two different approaches can be used. First, as shown in Eq. 4, total carbon emissions of of each zone throughout the year must remain below the maximum carbon allowance given by the carbon emissions rate limit $\times$ the total demand for each zone z. This requires that each zone must fulfill its limit independently. Second, as shown in Eq. 5, the constraint on maximum emissions can be used as a system-wide constraint, which mimics policies allowing for CO_2 emissions trading across the different zones.

$$\sum_{y\in\mathcal{G}}\sum_{t\in\mathcal{T}}(\epsilon_{y,z}^{CO_2} \times (\Theta_{y,t,z} + \Pi_{y,t,z})) \leq \epsilon_z^{max} \times \sum_{t\in\mathcal{T}} D_{t,z}, \qquad \forall z \in \mathcal{Z} \tag{4}$$

$$\sum_{z\in\mathcal{Z}}\sum_{y\in\mathcal{G}}\sum_{t\in\mathcal{T}}(\epsilon_{y,z}^{CO_2} \times (\Theta_{y,t,z} + \Pi_{y,t,z})) \leq \sum_{z\in\mathcal{Z}}\sum_{t\in\mathcal{T}}(\epsilon_z^{max} \times D_{t,z}), \tag{5}$$

Note that if the model is fully linear (e.g., $\mathcal{UC} = \emptyset$), the dual variable of the emissions constraints can be interpreted as the marginal CO_2 price per ton associated with the emissions target of ϵ_z^{max}. Depending on the constraint used, the there will be either a system-wide price or multiple zone-specific prices.

5.5 Accounting for Renewable Energy Requirements

In the case that the user wishes to model a renewable electricity mandate, renewable portfolio standard policy, or other constraint requiring a minimum share of energy from qualifying renewable energy resources, the model provides two approaches. First, Eq. 6 assumes each zone in the system must fulfill the minimum share of annual energy from qualifying resources independently. The total mandate is calculated as the the share, μ_z^{RE}, of the total demand on each zone that must be served by qualifying renewable energy resources, specified as those resources belonging to the set $\mathcal{RE}$. A second approach is to assume that the mandate, that can differ by zone, can be fulfilled at a system-wide scale (e.g., such as when trading of renewable energy credits is allowed across a region). In this case Eq. 7 is used as the constraint in place. Total demand that must be served by qualifying renewable resources correspond to the sum across the different mandates of all zones.

$$\sum_{y \in \mathcal{RE}} \sum_{t \in \mathcal{T}} \Theta_{y,t,z} \geq \mu_z^{RE} \times \sum_{t \in \mathcal{T}} D_{t,z}, \qquad \forall z \in \mathcal{Z} \tag{6}$$

$$\sum_{z \in \mathcal{Z}} \sum_{y \in \mathcal{RE}} \sum_{t \in \mathcal{T}} \Theta_{y,t,z} \geq \sum_{z \in \mathcal{Z}} \sum_{t \in \mathcal{T}} (\mu_z^{RE} \times D_{t,z}), \tag{7}$$

5.6 Accounting for Demand Balance

The demand balance constraint of the model (see Eq. 8) ensures that electricity demand is met at every hour in each zone. As shown in the constraint, electricity demand, $D_{t,z}$, at each hour and for each zone must be strictly equal to the sum of generation, $\Theta_{y,t,z}$, from thermal technologies ($\mathcal{H}$), dispatchable renewables ($\mathcal{D}$), non-dispatchable renewables ($\mathcal{ND}$), and hydro resources ($\mathcal{W}$). At the same time, pumped hydroelectric and electrical energy storage devices ($\mathcal{O}$) can discharge energy, $\Theta_{y,t,z}$ to help satisfy demand, while when these devices are charging, $\Pi_{y,t,z}$, they increase demand. For the case of shiftable demand ($\mathcal{DR}$), delaying demand, $\Pi_{y,t,z}$, decreases demand while satisfying delayed demand, $\Theta_{y,t,z}$, increases demand. Heat storage ($\mathcal{HO}$) can only increase demand directly by charging ,$\Pi_{y,t,z}$, since discharged heat is either sold or used for generation in NACCs. NACC technologies ($\mathcal{AN}$) also contribute with generation from both their normal cycle, $\Theta_{y,t,z}$, and by inputting heat to generate additional electricity via a topping cycle, equal to $\eta_{y,z}^{heat} \times (\sigma_{x,t,z} + \nu_{y,t,z})$. Price-responsive demand curtailment, $\Lambda_{s,t,z}$, also reduces demand. Finally, power flows into or out of a zone are considered in the demand balance equation for each zone. By definition, power flows leaving their reference zone are positive, thus the minus sign in the constraint Eq 8. At the same time losses due to power flows can only increase demand, and one-half of losses across a line linking two zones are attributed to each connected zone. The losses function $\beta_{l,t}(\cdot)$ will depend on the configuration used (see section 5.8). In addition, if distribution network zones are modeled (see Section 5.17), distribution losses reflecting power flows within each distribution zone, $\ell z, t$ are also applied.

$$\begin{aligned}
&\sum_{y \in \mathcal{H}} \Theta_{y,t,z} + \sum_{y \in \mathcal{D}} \Theta_{y,t,z} + \sum_{y \in \mathcal{ND}} \Theta_{y,t,z} + \sum_{y \in \mathcal{O}} (\Theta_{y,t,z} - \Pi_{y,t,z}) \\
&+ \sum_{y \in \mathcal{DR}} (-\Theta_{y,t,z} + \Pi_{y,t,z}) - \sum_{y \in \mathcal{HO}} \Pi_{y,t,z} + \sum_{y \in \mathcal{W}} \Theta_{y,t,z} \\
&+ \sum_{y \in \mathcal{AN}} (\Theta_{y,t,z} + \eta_{y,z}^{heat} (\sigma_{x,t,z} + \nu_{y,t,z})) \\
&+ \sum_{s \in \mathcal{S}} \Lambda_{s,t,z} - \sum_{l \in \mathcal{L}} (\varphi_{l,z}^{map} \times \Phi_{l,t}) - \frac{1}{2} \sum_{l \in \mathcal{L}} (\mid \varphi_{l,z}^{map} \mid \times \beta_{l,t}(\cdot)) - \ell_{z,t} = D_{t,z} \qquad \forall z \in \mathcal{Z}, \forall t \in \mathcal{T}
\end{aligned} \tag{8}$$

Note that if the model is fully linear (e.g., $\mathcal{UC} = \emptyset$), the dual variable of the demand balance constraint can be interpreted as the hourly price of electricity at each zone z. In addition, if unit commitment and discrete capacity expansion decisions are considered, GenX is programmed to allow the user to optionally output hourly prices by solving the model a second time with all integer decisions fixed to equal outcomes of the initial MILP solution, using the dual variable of the demand balance constraint in this second solution as the hourly price. This option does increase total run-time due to the second solution stage, but this solution step is much faster than the original solution time and can be tractable. However, the default setting for MILP problems is to not provide dual variables in this manner, unless the user overrides this default setting to request hourly prices as an output.

5.7 Accounting for Transmission and Network Expansion Between Zones

Different approaches for power flow calculations can be used. The first approach uses a "transport method," where Eq. 9 limits the maximum power flow, $\Phi_{l,t}$, on each line to be less than or equal to the line's maximum power transfer capacity, φ_l^{max} When network expansion configuration is activated, Eq. 9 is replaced by Eq. 10 for eligible lines in the set $\mathcal{E}$, reflecting that power flows must be less than the initial line transfer capacity plus any transmission capacity added on that line. The additional transmission capacity, $\triangle\varphi_l^{max}$, is constrained by a maximum allowed reinforcement, $\overline{\triangle\varphi_l^{max}}$, for each line, as is shown in Eq. 11.

$$
\begin{aligned}
& -\varphi_l^{max} \leq \Phi_{l,t} \leq \varphi_l^{max}, && \forall l \in (\mathcal{L} \setminus \mathcal{E}), \forall t \in \mathcal{T} && (9)\\
& -(\varphi_l^{max} + \triangle\varphi_l^{max}) \leq \Phi_{l,t} \leq (\varphi_l^{max} + \triangle\varphi_l^{max}), && \forall l \in \mathcal{E}, \forall t \in \mathcal{T} && (10)\\
& \triangle\,\varphi_l^{max} \leq \overline{\triangle\varphi_l^{max}}, && \forall l \in \mathcal{E}, \forall t \in \mathcal{T} && (11)
\end{aligned}
$$

The second approach uses a DC optimal power flow (DC OPF) method [35] to govern power flows across all lines. Eq. 12 establishes the relationship between the power flow on each line, the line's parameters for voltage, φ_l^{volt}, impedance, φ_l^{ohm}, and the difference between the angle, $\vartheta_{z,t}$, present on each zone/node connected by the line. Eq. 13 limits the angle difference between the nodes connected by a line to a maximum angle, φ_l^{θ}, to respect network stability requirements. Finally, Eq. 14 set the angle of zone 1 of the system as the reference node and equal to zero.

$$
\begin{aligned}
& \Phi_{l,t} = \frac{(\varphi_l^{volt})^2}{\varphi_l^{ohm}} \sum_{z \in \mathcal{Z}} (\varphi_{l,z}^{map} \times \vartheta_{z,t}), && \forall l \in \mathcal{L}, \forall t \in \mathcal{T} && (12)\\
& -\varphi_l^{\theta} \leq \sum_{z \in \mathcal{Z}} (\varphi_{l,z}^{map} \times \vartheta_{z,t}) \leq \varphi_l^{\theta}, && \forall l \in \mathcal{L}, \forall t \in \mathcal{T} && (13)\\
& \vartheta_{1,t} = 0, && \forall t \in \mathcal{T} && (14)
\end{aligned}
$$

5.8 Accounting for Transmission Losses

Transmission losses due to power flows can be accounted for in different ways. As shown in Eq. 15 the the first option is to neglect losses entirely, setting the value of the losses function to zero for all lines at all hours. The second option is to assume that losses are a fixed percentage, φ_l^{loss}, of the magnitude of power flow on each line, $\mid \Phi_{l,t} \mid$ (e.g., losses are a linear function of power flows). Finally, the third option is to calculate losses, $\ell_{l,t}$ by approximating a quadratic-loss function using a piecewise-linear function with total number of segments equal to the size of the set $\mathcal{M}$.

$$\beta_{l,t}(\cdot) = \begin{cases} 0 & \text{if losses. } 0 \\ \varphi_l^{loss} \times \mid \Phi_{l,t} \mid & \text{if losses. } 1 \text{ ,} \\ \ell_{l,t} & \text{if losses. } 2 \end{cases} \qquad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{15}$$

For the second option, a simple absolute value approximation is utilized to calculate the magnitude of the power flow on each line (reflecting the fact that negative power flows for a line linking nodes i and j represents flows from node j to i and causes the same magnitude off losses as an equal power flow from i to j). Eq. 16 linearizes this absolute value function, such that the flow in the line must be equal to the subtraction of the auxiliary variable for flow in the positive direction, $\Phi_{l,t}^{+}$, and the auxiliary variable for flow in the negative direction, $\Phi_{l,t}^{+}$, of the line. Then, the magnitude of the flow is calculated as the sum of the two auxiliary variables as shown in Eq. 17. In order to minimize the creation of "phantom losses" in the system, Eq. 18 and Eq. 19 limit the value of the auxiliary variables to the maximum transmission capacity of the line.

$$\Phi_{l,t} = \Phi_{l,t}^{+} - \Phi_{l,t}^{-}, \qquad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{16}$$

$$\mid \Phi_{l,t} \mid = \Phi_{l,t}^{+} + \Phi_{l,t}^{-}, \qquad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{17}$$

$$\Phi_{l,t}^{+} \leq \varphi_l^{max}, \qquad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{18}$$

$$\Phi_{l,t}^{-} \leq \varphi_l^{max}, \qquad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{19}$$

For the third option, losses are calculated as a piecewise-linear approximation of a quadratic loss function. In order to do this, we represent the absolute value of the line flow variable by the sum of positive stepwise flow variables ($\S_{m,l,t}^{+}, \S_{m,l,t}^{-}$), associated with each partition of line losses computed using the corresponding linear expressions. This can be understood as a segmentwise linear fitting (or first order approximation) of the quadratic losses function. Generally, the model includes constraints Eq. 20–Eq. 29. Eq. 20 provides the expression of linearized losses, which are computed as the accumulated sum of losses for each linear stepwise segment of the approximated quadratic function, including both positive domain and negative domain segments; Eq. 21 ensures that the stepwise variables do not exceed the maximum size per segment. The slope and maximum size for each segment are calculated as per the method in [37].

$$\ell_{l,t} = \frac{\varphi_l^{ohm}}{(\varphi_l^{volt})^2}\Bigg(\sum_{m\in\mathcal{M}}(S^+_{m,l}\times \S^+_{m,l,t} + S^-_{m,l}\times \S^-_{m,l,t})\Bigg), \qquad \forall l\in\mathcal{L},\forall t\in\mathcal{T}$$

Where:

$$S^+_{m,l} = \frac{2+4\times\sqrt{2}\times(m-1)}{1+\sqrt{2}\times(2\times M-1)}(\varphi_l^{max}+\overline{\triangle\varphi_l^{max}}) \qquad \forall m\in[1\colon M], l\in\mathcal{L}$$

$$S^-_{m,l} = \frac{2+4\times\sqrt{2}\times(m-1)}{1+\sqrt{2}\times(2\times M-1)}(\varphi_l^{max}+\overline{\triangle\varphi_l^{max}}) \qquad \forall m\in[1\colon M], l\in\mathcal{L} \tag{20}$$

$$\S^+_{m,l,t}, \S^-_{m,l,t} <= \overline{\S_{m,l}} \qquad \forall m\in[1:M], l\in\mathcal{L}, t\in\mathcal{T}$$

Where:

$$\overline{\S_{l,z}} = \begin{cases} \frac{(1+\sqrt{2})}{1+\sqrt{2}\times(2\times M-1)}(\varphi_l^{max}+\overline{\triangle\varphi_l^{max}}) & \text{if m=1} \\ \\ \frac{2\times\sqrt{2}}{1+\sqrt{2}\times(2\times M-1)}(\varphi_l^{max}+\overline{\triangle\varphi_l^{max}}) & \text{if m > 1} \end{cases}, \tag{21}$$

Eq. 22 and Eq. 23 ensure that the sum of auxiliary segment variables ($m \geq 1$) minus the “zero” segment (which allows values to go into the negative domain) from both positive and negative domains must total the actual power flow across the line.

$$\sum_{m\in[1:M]}(\S^+_{m,l,t}) - \S^+_{0,l,t} = \Phi_{l,t}, \qquad \forall l\in\mathcal{L},\forall t\in\mathcal{T} \tag{22}$$

$$\sum_{m\in[1:M]}(\S^-_{m,l,t}) - \S^-_{0,l,t} = -\Phi_{l,t}, \qquad \forall l\in\mathcal{L},\forall t\in\mathcal{T} \tag{23}$$

Eq. 24–27 ensure that auxiliary segments variables do not exceed maximum value per segment and that they are filled in order; i.e., one segment cannot be non-zero unless prior segment is at its maximum value. Eq. 28 and Eq. 29 are binary constraints to deal with absolute value of power flow on each line. If the flow is positive, $\S^+_{0,l,t}$ must be zero; if flow is negative, $\S^+_{0,l,t}$ must be positive and takes on value of the full negative flow, forcing all $\S^+_{m,l,t}$ other segments ($m \geq 1$) to be zero. Conversely, if the flow is negative, $\S^-_{0,l,t}$ must be zero; if flow is positive, $\S^-_{0,l,t}$ must be positive and takes on value of the full positive flow, forcing all $\S^-_{m,l,t}$ other segments ($m \geq 1$) to be zero. Requiring segments to fill in sequential order and binary variables to ensure variables reflect the actual direction of power flows are both necessary to eliminate “phantom losses” from the solution, which can occur when unit commitment decisions are employed for some generators. In the case where discrete unit commitment decisions are considered, increasing demand by increasing losses incurs additional supply costs but may avoid greater costs associated with discrete start-up decisions. The solution may therefore involve “phantom losses” that are not associated with actual power flows (see [38]). If the model is fully linear (e.g., $\mathcal{UC} = \emptyset$), Eq. 24–29 are automatically omitted to reduce dimensionality.

$$\S^{+}_{m,l,t} <= \overline{\S_{m,l}} \times ON^{+}_{m,l,t}, \qquad \forall m \in [1:M], \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{24}$$

$$\S^{-}_{m,l,t} <= \overline{\S_{m,l}} \times ON^{-}_{m,l,t}, \qquad \forall m \in [1:M], \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{25}$$

$$\S^{+}_{m,l,t} \geq ON^{+}_{m+1,l,t} \times \overline{\S_{m,l}}, \qquad \forall m \in [1:M], \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{26}$$

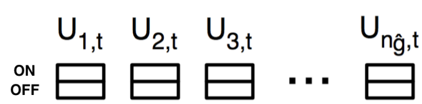
$$\S^{-}_{m,l,t} \geq ON^{-}_{m+1,l,t} \times \overline{\S_{m,l}}, \qquad \forall m \in [1:M], \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{27}$$

$$\S^{+}_{0,l,t} \leq \varphi^{max}_{l} \times (1 - ON^{+}_{1,l,t}), \qquad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{28}$$

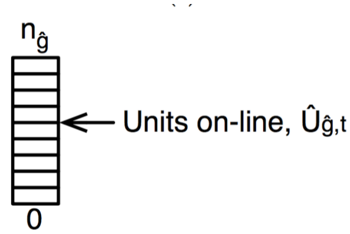
$$\S^{-}_{0,l,t} \leq \varphi^{max}_{l} \times (1 - ON^{-}_{1,l,t}), \qquad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{29}$$

5.9 Accounting for Unit Commitment

As already mentioned, commitment and cycling (start-up, shut-down) of thermal generators is accounted for using the integer clustering technique developed in [17–19]. In a typical binary unit commitment formulation, each unit is either on or off [20, 34]. With the clustered unit commitment formulation, generators are clustered by type and zone, and the integer commitment state variable for each cluster varies from zero to the number of units in the cluster, $(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}) = n_{\hat{g}}$. As shown in Fig. 4, this approach replaces the large set of binary commitment decisions and associated constraints, which scale directly with the number of individual units, with a smaller set of integer commitment states and constraints, one for each cluster y. The dimensionality of the problem thus scales with the number of units of a given type in each zone, rather than by the number of discrete units, significantly improving computational efficiency. However, this method entails the simplifying assumption that all clustered units have identical parameters (e.g., capacity size, ramp rates, heat rate) and that all committed units in a given time step t are operating at the same power output per unit (see [17–19, 23] for detailed discussion of this method and the magnitude of resulting abstraction errors in different contexts).



(a) Unit commitment with separate binary variables



(b) Unit commitment with clustered integer variable

Figure 4: Unit Commitment Approaches [18]

Note that in a capacity expansion problem, the number of units that may be built (e.g., are eligible for capacity expansion) can significantly exceed the number of units in the final solution. For realistic-size systems, capacity expansion with binary unit commitment decisions is typically not computationally tractable unless the dimensionality is reduced by selecting a significantly reduced set of representative hours, rather than modeling a full year [20]. However, the integer clustering formulation implemented in GenX is flexible enough to represent binary unit commitment at the individual generator level if desired, simply by setting the maximum number of generators per cluster to 1 for each generator type and zone.

In addition, it is possible to configure GenX to model a linear relaxation of the discrete unit commitment decisions and related constraints. This is accomplished by replacing the integer unit commitment and capacity addition variables with continuous variables, subject to the same set of constraints (e.g., Equations 30–39). That is, the feasible solution set must be contained in the convex hull formed by the integer commitment decisions and constraints described in this section. This option significantly improves computational tractability, but entails a reduction in accuracy as final solutions ignore the discrete nature of real-world generator construction and unit commitment decisions. By still employing the full set of unit commitment constraints, however, this linear relaxation produces more accurate results than a simple linear economic dispatch formulation that entirely ignores unit commitment decisions. This tradeoff may be acceptable for some problems, particularly when it allows an increase in model resolution in one or more other dimensions more relevant to the questions at hand.

This set of options for representing operational constraints related to generator unit commitment decisions reflects the highly configurable nature of the GenX model and allows the user to balance tradeoffs between detail and abstraction in both the chronology and operational dimensions (see Figure 3).

5.9.1 Startup and Shutdown Events

As shown in Eq. 30–32, the maximum number of plants that can be committed, $\upsilon_{y,t,z}$, started, $\chi_{y,t,z}$, or shutdown, $\zeta_{y,t,z}$, at any given time, is given by the net installed generation capacity of the cluster y in zone z. Eq. 33 sets the relationship between the commitment state of the cluster, the startups, and shutdowns. For example, during hour $t-1$ the cluster y has a total of 5 units installed and has 3 of them committed (generating), $\upsilon_{y,t-1,z} = 3$. Then, if at hour t the cluster y needs a 4th unit online ($\upsilon_{y,t,z} = 4$), the startup variable $\chi_{y,t,z}$ will have to take a value of 1, so Eq. 35 is fulfilled ($4 = 3 + 1 - 0$). As described in the objective function (Eq. 1), this startup decision will incur a startup cost $\pi^{START}_{y,z}$ for every plant that it is started in cluster y.

$$\upsilon_{y,t,z} \leq \frac{\overline{\Delta_{y,z}}}{\Omega^{size}_{y,z}} + \Omega_{y,z} - \Delta_{y,z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{30}$$

$$\chi_{y,t,z} \leq \frac{\overline{\Delta_{y,z}}}{\Omega^{size}_{y,z}} + \Omega_{y,z} - \Delta_{y,z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{31}$$

$$\zeta_{y,t,z} \leq \frac{\overline{\Delta_{y,z}}}{\Omega^{size}_{y,z}} + \Omega_{y,z} - \Delta_{y,z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{32}$$

$$\upsilon_{y,t,z} = \upsilon_{y,t-1,z} + \chi_{y,t,z} - \zeta_{y,t,z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{33}$$

5.9.2 Minimum and Maximum Output

Eq. 35 and 34 require that clustered thermal generators' output, $\Theta_{y,t,z}$, must be kept between maximum and minimum output levels. The maximum output, $\rho^{max}_{y,t,z}$, and minimum stable output, $\rho^{min}_{y,z}$ are specified for each resource type, y in each zone z, as a per unit share of the rated capacity size for each generator of that resource type, given by $\Omega^{size}_{y,z}$. Eq. 35- 34 then specify that the aggregate maximum and minimum power output for the cluster in time t is then proportional to the number of committed generators, $\upsilon_{y,t,z}$, in the cluster.

$$\Theta_{y,t,z} \geq \rho^{min}_{y,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{34}$$

$$\Theta_{y,t,z} \leq \rho^{max}_{y,t,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{35}$$

5.9.3 Ramping Limits

Ramping limits capture constraints on how fast thermal units can adjust their power output in a given time step (e.g., change in power output per hour), reflecting constraints that limit the rate of change in output to avoid related thermal and mechanical stresses on generator equipment. For clusters of thermal generators, the hourly change in output must be constrained to reflect limits on the rate of change in the output of committed (online) generators, as well changes in aggregate output of the cluster reflecting start-up or shut-down of individual units in the cluster within the time step.

Eq. 36 shows the ramp-down constraint. The difference in power output between two consecutive hours ($\Theta_{y,t-1,z} - \Theta_{y,t,z}$) for cluster y in zone z must be less than or equal to the feasible ramp-down rate, $\kappa^{down}_{y,z}$ (expressed in per unit terms), times the unit size for generators in that cluster, $\Omega^{size}_{y,z}$, times the number of committed units in that time step, $\upsilon_{y,t,z}$. This portion of the constraint reflects only the allowed average change from committed units. However, during the same hour, additional units may start, $\chi_{y,t,z}$, bringing new units in the cluster online. Thus, Eq. 36 must also account for the impact of any newly committed units, which must operate at or above their minimum stable output level, on the feasible change in aggregate output for the cluster during that time step. This is accomplished by subtracting the newly started units from the number of committed units in hour t in the first part of the equation, such that only units that were committed at the beginning of the time period (and not newly started units) can change their power output at or below the allowed per unit ramp rate. In addition, it is necessary to subtract the number of started units times their individual unit size and minimum stable output, $\rho^{min}_{y,z}$, from the total allowed change in aggregate output for the cluster, since these units cannot be shutdown during the same hour and must operate at or above their minimum stable output. Finally, the ramp-down constraint must account for units that are being shutdown during the hour t, which may allow a larger change in aggregate output for the cluster. Thus, the minimum between the maximum output and the maximum between the minimum stable output or the maximum ramp-down rate, times the plant size and the number of shutdowns during hour t is added to the ramp-down capacity. In other words, an individual unit shutting down can result in a change in aggregate output for the cluster equal to the greater of either its minimum stable output level or its hourly ramp rate (the inclusion of $\rho^{max}_{y,t,z}$ is intended to apply only in cases when the per unit ramp rate is > 1.0, and is redundant in all other cases).

$$\Theta_{y,t-1,z} - \Theta_{y,t,z} \leq \kappa_{y,z}^{down} \times \Omega_{y,z}^{size} \times (\upsilon_{y,t,z} - \chi_{y,t,z})$$
$$- \rho_{y,z}^{min} \times \Omega_{y,z}^{size} \times \chi_{y,t,z}$$
$$+ \min(\rho_{y,t,z}^{max}, \max(\rho_{y,z}^{min}, \kappa_{y,z}^{down})) \times \Omega_{y,z}^{size} \times \zeta_{y,t,z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \qquad (36)$$

Eq. 37 presents the constraint on ramping in the upward direction. As this equation illustrates, the difference between aggregate power output between any two consecutive hours ($\Theta_{y,t,z} - \Theta_{y,t-1,z}$) for cluster y in zone z, must be less than or equal to the feasible ramp-up rate, $\kappa_{y,z}^{up}$ (expressed in per unit terms), times the unit size for generators in that cluster, $\Omega_{y,z}^{size}$, times the number of committed units in that time step, $\upsilon_{y,t,z}$. Once again, this portion of the constraint only accounts for average change in output from committed units without taking into account start-ups and shut-downs. Thus, newly started units, $\chi_{y,t,z}$, must be subtracted from the committed units to which hourly ramp limits apply. Additionally, an extra term must be added accounting for the increase in aggregate power output from the cluster due to newly started units, which can increase output up to the minimum between their maximum output and the minimum between their minimum stable output and the ramp-up rate, times the plan size and the number of started units. Finally, shutdowns in the cluster at hour t are taken into account by subtracting the minimum stable output, $\rho_{y,z}^{min}$, times the plant size and the number of shutdowns in the cluster.

$$\Theta_{y,t,z} - \Theta_{y,t-1,z} \leq \kappa_{y,z}^{up} \times \Omega_{y,z}^{size} \times (\upsilon_{y,t,z} - \chi_{y,t,z})$$
$$+ \min(\rho_{y,t,z}^{max}, \max(\rho_{y,z}^{min}, \kappa_{y,z}^{up})) \times \Omega_{y,z}^{size} \times \chi_{y,t,z}$$
$$- \rho_{y,z}^{min} \times \Omega_{y,z}^{size} \times \zeta_{y,t,z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \qquad (37)$$

5.9.4 Minimum Up and Down Times

Thermal generators subject to unit commitment decisions must also respect minimum "up time" and "down time" constraints reflecting limits on the period of time between when a unit starts-up and when it can be shut-down again, and vice versa. Eq. 38 ensures that the number of units online in time t, equals or exceeds the total number of start-ups during the proceeding $\tau_{y,z}^{up}$ time steps, where $\tau_{y,z}^{up}$ is the minimum up time for units in cluster y in zone z. Eq. 39 likewise ensures that the number of units offline in time t is equal to or greater than the total number of shut-downs during the proceeding $\tau_{y,z}^{down}$ time steps, where $\tau_{y,z}^{down}$ is the minimum down time for units in cluster y in zone z.

$$\upsilon_{y,t,z} \geq \sum_{\hat{t}=t-\tau_{y,z}^{up}}^{t} \chi_{y,\hat{t},z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \qquad (38)$$

$$(\frac{\overline{\Delta_{y,z}}}{\Omega_{y,z}^{size}} + \Omega_{y,z} - \Delta_{y,z}) - \upsilon_{y,t,z} \geq \sum_{\hat{t}=t-\tau_{y,z}^{down}}^{t} \zeta_{y,\hat{t},z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \qquad (39)$$

5.10 Non-Clustered Thermal Technologies: Operational Requirements

Thermal units for which unit commitment constraints do not apply (e.g., $y \in (\mathcal{H} \setminus \mathcal{UC})$ face a simplified set of constraints given by Eq. 40–43. For these generators, investment and dispatch decisions are considered continuous variables. In this case, hourly changes in output (ramps down and ramps up) are constrained by Eq. 40 and 41 to be less than the maximum ramp rates ($\kappa_{y,z}^{down}$ and $\kappa_{y,z}^{up}$) in per unit terms times the total installed capacity of technology y. In addition, the total power output for technology y must always operate between the minimum stable output, $\rho_{y,z}^{min}$, and the maximum output, $\rho_{y,t,z}^{max}$, expressed in per unit terms, times the total installed capacity of technology y (Eq. 42 and 43).

$$\Theta_{y,t-1,z} - \Theta_{y,t,z} \leq \kappa_{y,z}^{down} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in (\mathcal{H} \setminus \mathcal{UC}), \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{40}$$

$$\Theta_{y,t,z} - \Theta_{y,t-1,z} \leq \kappa_{y,z}^{up} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in (\mathcal{H} \setminus \mathcal{UC}), \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{41}$$

$$\Theta_{y,t,z} \geq \rho_{y,z}^{min} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in (\mathcal{H} \setminus \mathcal{UC}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{42}$$

$$\Theta_{y,t,z} \leq \rho_{y,t,z}^{max} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in (\mathcal{H} \setminus \mathcal{UC}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{43}$$

5.11 Accounting for Renewable Resources

Power output from variable renewable energy (VRE) resources, including wind and solar energy and run-of-river hydroelectric power, are treated as a function of each technology's hourly capacity factor (or availability factor), in per unit terms, and the total capacity deployed. As the available solar insolation, wind speed, or streamflow for a given VRE resource varies over time, the capacity factor ($\rho_{y,t,z}^{max}$) reflects the maximum possible power output in time t, expressed in per unit terms, and is specified exogenously for each resource type y and location z. For dispatchable VRE resources ($y \in \mathcal{D}$), power output can be curtailed if desired, and thus hourly power output $\Theta_{y,t,z}$ must be less than or equal to the the hourly capacity factor times the installed capacity of that resource, as depicted in Eq. 44. This adds the possibility of introducing VRE curtailment as an extra degree of freedom to guarantee that generation exactly meets hourly demand. On the other hand, Eq. 45 shows that for non-dispatchable renewable resources ($y \in \mathcal{ND}$, e.g., generally distributed VRE generators that do not receive dispatch signals), output must exactly equal the available capacity factor times the installed capacity, not allowing for curtailment.

$$\Theta_{y,t,z} \leq \rho_{y,t,z}^{max} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{D}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{44}$$

$$\Theta_{y,t,z} = \rho_{y,t,z}^{max} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{ND}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{45}$$

5.12 Accounting for Storage Technologies

Different storage technologies, including pumped-hydro storage, mechanical storage devices, or electro-chemical storage batteries of various types (e.g., $y \in \mathcal{O}$) are all parametrized by their "single-trip" charging and discharging efficiencies ($\eta_{y,z}^{up}$ and $\eta_{y,z}^{down}$, respectively), self-discharge rate (per hour, $\eta_{y,z}^{loss}$), and a fixed power to energy ratio, $\mu_{y,z}^{stor}$. This ratio captures the energy storage capacity (MWh) per unit of power charging/discharging capacity (MW), and can be considered the inverse of the numbers of hours it would take to fully discharge a full state of charge for a given power capacity of technology y (e.g., $\mu_{y,z}^{stor} = 0.2$ corresponds to an energy storage capacity suitable for five hours of discharging at maximum rate of power output). While the power to energy ratio for storage systems is considered fixed in the current implementation of GenX, multiple types of storage devices can be modeled with different discrete power to energy ratios, if desired. Note also that in GenX, rated power capacity is treated as the AC injection and withdrawal capacity at the interface with the grid. Charging and discharging efficiencies are thus applied "up-stream" of the injection/withdrawal interface, and reflect factors such as single-trip AC-DC inverter losses (if relevant), other balance of system losses, and charge/discharge conversion losses. Storage devices are then governed by a set of constraints which track the state of charge or stored energy level over time steps and constrain the state of charge and the charging and discharging power rates based on installed capacity.

Eq. 46 represents the state of charge or storage energy level of the storage capacity of technology y across time, reflecting: the state of charge in the previous time step, $t-1$; less the amount of energy discharged in period t, which reflects the amount of power injected ($\Theta_{y,t,z}$) and the storage discharge efficiency ($\eta_{y,z}^{down}$); plus the amount of energy added to the storage system (charged) in period t, which reflects the amount of power withdrawn from the grid ($\Pi_{y,t,z}$) and the charging efficiency ($\eta_{y,z}^{up}$); and finally, less the amount of energy losses due to self discharge ($\eta_{y,z}^{loss}$).

The maximum energy stored (MWh) at any given time t must be less than or equal to the energy storage capacity, as governed by Eq. 47. Notice that the investment decision in all technologies reflects installed power capacity (MW), which in the case of storage devices, reflects the charge/discharge rates. Thus storage energy capacity is calculated based on the power to energy ratio ($\mu_{y,z}^{stor}$) and the net installed power capacity ($\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}$) for the particular storage technology.

Charge rates are constrained by both the net installed power capacity (increased by the charge efficiency) (see Eq. 48) and by the remaining energy capacity in the current time step, given by the installed energy storage capacity minus the current storage level (see Eq. 49). In the same way, discharge rates are constrained by either the net installed capacity (decreased by the discharge efficiency) (see Eq. 50) or by the current energy storage level (see Eq. 51), whichever is more constraining.

Finally, note that since storage investment decisions are modeled as continuous, storage capacity represents the aggregated capacity of a number of similar storage units. Thus, charging and discharging are not mutually exclusive in any give time step (e.g., some storage units may be charging and some discharging, if needed). However, the total power devoted to charging and discharging combined in any given time step t must remain less than or equal to the total installed capacity as shown in Eq. 52.

$$\Gamma_{y,t,z} = \Gamma_{y,t-1,z} - \left(\frac{\Theta_{y,t,z}}{\eta_{y,z}^{down}}\right) + (\eta_{y,z}^{up} \times \Pi_{y,t,z}) - (\eta_{y,z}^{loss} \times \Gamma_{y,t,z}), \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{46}$$

$$\Gamma_{y,t,z} \leq \frac{1}{\mu_{y,z}^{stor}} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{47}$$

$$\Pi_{y,t,z} \leq \left(\frac{1}{\eta_{y,z}^{up}}\right) \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{48}$$

$$\Pi_{y,t,z} \leq \frac{1}{\mu_{y,z}^{stor}} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}) - \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{49}$$

$$\Theta_{y,t,z} \leq \eta_{y,z}^{down} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{50}$$

$$\Theta_{y,t,z} \leq \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{51}$$

$$\left(\frac{\Theta_{y,t,z}}{\eta_{y,z}^{down}}\right) + (\eta_{y,z}^{up} \times \Pi_{y,t,z}) \leq (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{52}$$

5.13 Accounting for Demand Side Resources

Two different demand side resources are considered in the present implementation. The first type is demand side management (DSM), which represents demand which may be shifted or deferred in time (i.e., some thermal loads, electric vehicle charging, water pumping, etc.). As implemented, DSM is characterized by the maximum percentage of demand at hour t that can be shifted, $\mu_{y,z}^{DMS}$, and the maximum time before this demand must be satisfied, $\overline{\tau_{y,z}}$. Multiple segments of deferrable or shiftable demand can be specified for each zone by setting different values of $\mu_{y,z}^{DMS}$ and $\overline{\tau_{y,z}}$ for each segment.

Eq. 53–55 describe the DSM operational constraints. The amount of deferred demand remaining to be served, $\Gamma_{y,t,z}$, depends on the amount in the previous hour $t-1$, minus the served energy during hour t, $\Theta_{y,t,z}$, plus the demand that has been deferred during the current hour, $\Pi_{y,t,z}$ (see Eq. 53). At any given hour, the maximum amount of demand that can be shifted or deferred, $\Pi_{y,t,z}$, correspond to a fraction, $\mu_{y,z}^{DMS}$, of the demand in that time period, $D_{t,z}$ (see Eq. 54). Shifted demand must then be served within a fixed period of time, as shown in Eq. 55. This is done by forcing the sum of DSM demand satisfied in the following τ hours (e.g., $t+1$ to $t+\tau$) to be greater than or equal to the level of DSM energy deferred during time t.

$$\Gamma_{y,t,z} = \Gamma_{y,t-1,z} - \Theta_{y,t,z} + \Pi_{y,t,z}, \qquad \forall y \in \mathcal{DR}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{53}$$

$$\Pi_{y,t,z} \leq \mu_{y,z}^{DMS} \times D_{t,z}, \qquad \forall y \in \mathcal{DR}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{54}$$

$$\sum_{\hat{t}=t+1}^{t+\overline{\tau_{y,z}}} \Theta_{y,\hat{t},z} \geq \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{DR}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{55}$$

The second mechanism is demand response or price-responsive demand curtailment. Under this mechanism different segments s of the hourly demand represent aggregations of consumers with different marginal value of electricity consumption, given by n_s^{slope}. Thus, for a penalty or price equal to n_s^{slope}, consumers in segment s would be willing to curtail their demand in a given time period t. Thus, in each time step, t, the amount of curtailed demand or non-served energy for each consumer segment, $\Lambda_{s,t,z}$, must be less than or equal to the fraction of demand, n_s^{size}, that belongs to segment s times the hourly demand $D_{t,z}$ (see Eq. 56).

$$\Lambda_{s,t,z} \leq (n_s^{size} \times D_{t,z}), \qquad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{56}$$

5.14 Accounting for NACC and Heat Storage

Nuclear Air-Brayton Combined Cycle (NACC) presents a new operational paradigm: base load nuclear generation with additional peaking capabilities provided by a "topping cycle" via extra heat added to the Brayton cycle from either heat storage or combustion of natural gas. In addition, excess electrical energy from the nuclear generator can be converted to high temperature heat via resistive heaters and stored in thermal storage systems (e.g., Resistance-Heated Heat Storage). This stored thermal energy can then be used to supply heat for the topping cycle of the NACC generator or sold into the local heat market (e.g., for industrial process heat). In GenX, NACC units can be treated as discrete units subject to unit commitment constraints using the integer clustering method described above (e.g., $y \in (\mathcal{AN} \cap \mathcal{UC})$), or they can be modeled using a continuous capacity and dispatch decisions (e.g., $y \in (\mathcal{AN} \setminus \mathcal{UC})$).

When modeled as continuous decisions, the aggregate base load output of the NACC generators, $\Theta_{y,t,z}$, must always operate at the rated capacity installed (See Eq. 57). Peaking generation from the topping cycle is then a continuous decision and can be fueled via combination of two inputs (see 8). First, heat can be supplied by combustion of natural gas, $\nu_{y,t,z}$, at cost equal to the a local price of natural gas, π_z^{HEAT}, subject to a topping cycle conversion efficiency, $\eta_{y,z}^{heat}$. Second, heat can be supplied from heat storage, $\sigma_{x,t,z}$, subject to the same conversion efficiency, $\eta_{y,z}^{heat}$. Heat input for peaking generation cannot exceed the net installed NACC baseload capacity times $\mu_{y,z}^{heat}$, a parameter reflecting the ratio of peaking capacity per unit of baseload capacity for the NACC system, as shown in Eq. 58.

$$\Theta_{y,t,z} = (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in (\mathcal{AN} \setminus \mathcal{UC}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{57}$$

$$\sigma_{x,t,z} + \nu_{y,t,z} \leq \mu_{y,z}^{heat}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in (\mathcal{AN} \setminus \mathcal{UC}), \forall x \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{58}$$

When NACC systems are deployed subject to discrete investment and unit commitment decisions, the base load output, $\Theta_{y,t,z}$, must operate at exactly the rated capacity of the total number of committed units in the cluster (See Eq. 59). Peak generation can be achieved via input of heat from thermal storage or from natural gas combustion, as above, while total heat input for peaking generation cannot exceed the maximum peak capacity, $\mu_{y,z}^{heat}$, of the committed units in the cluster, as shown in Eq. 60.

$$\Theta_{y,t,z} = \rho_{y,t,z}^{max} \times \Omega_{y,z}^{size} \times \upsilon_{y,t,z}, \qquad \forall y \in (\mathcal{AN} \cap \mathcal{UC}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{59}$$

$$\sigma_{x,t,z} + \nu_{y,t,z} \leq \mu_{y,z}^{heat} \times \Omega_{y,z}^{size} \times \upsilon_{y,t,z}, \qquad \forall y \in (\mathcal{AN} \cap \mathcal{UC}), \forall x \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{60}$$

Resistance-heated heat storage systems coupled to NACC generators operate more or less as any other storage technology, as shown in Eq. 61–67, except that, as shown in Eq. 68, the total heat discharged from thermal storage is directed to use in peaking generation at NACC units, $\sigma_{y,t,z}$, or sold in the heat market, $\epsilon_{y,t,z}$. Furthermore, heat sold in the heat market must be less or equal to the heat demand, $H_{t,z}$, at any given time (See Eq. 69).

$$\Gamma_{y,t,z} = \Gamma_{y,t-1,z} - \left(\frac{\Theta_{y,t,z}}{\eta_{y,z}^{down}}\right) + (\eta_{y,z}^{up} \times \Pi_{y,t,z}) - (\eta_{y,z}^{loss} \times \Gamma_{y,t,z}), \qquad \forall y \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{61}$$

$$\Gamma_{y,t,z} \leq \frac{1}{\mu_{y,z}^{stor}} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{62}$$

$$\Pi_{y,t,z} \leq \left(\frac{1}{\eta^{up}_{y,z}}\right) \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \quad (63)$$

$$\Pi_{y,t,z} \leq \frac{1}{\mu^{stor}_{y,z}} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}) - \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \quad (64)$$

$$\Theta_{y,t,z} \leq \eta^{down}_{y,z} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \quad (65)$$

$$\Theta_{y,t,z} \leq \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \quad (66)$$

$$\left(\frac{\Theta_{y,t,z}}{\eta^{down}_{y,z}}\right) + (\eta^{up}_{y,z} \times \Pi_{y,t,z}) \leq (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \quad (67)$$

$$\epsilon_{y,t,z} + \sigma_{y,t,z} = \Theta_{y,t,z}, \qquad \forall y \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \quad (68)$$

$$\sum_{y \in \mathcal{HO}} \epsilon_{y,t,z} \leq H_{t,z}, \qquad \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \quad (69)$$

5.15 Accounting for Hydro Reservoir Resources

Hydroelectric generators with water storage reservoirs ($y \in \mathcal{W}$) are effectively modeled as energy storage devices that cannot charge and instead receive exogenous inflows to their storage reservoirs, reflecting streamflow inputs. Reservoir hydro resources are thus parametrized by their generation efficiency, $\eta^{down}_{y,z}$, their power to energy ratio, $\mu^{stor}_{y,z}$, which (in combination with the capacity decision variable) determines the total reservoir size in energy terms, and their initial energy level in the reservoir at the start of the simulation period, $w^{level}_{y,z}$, expressed as fraction of the total energy capacity of the reservoir. Additionally, energy inflows to the reservoir at every hour, as a fraction of the total energy capacity, are introduced by the hourly parameter, $\rho^{max}_{y,t,z}$.

Reservoir hydro systems are then governed by the following constraints. First, Eq. 70 represents the energy level of the reservoir resource y in time t, which the sum of the reservoir level in the previous time step, $t-1$, less the amount of electricity generated, $\Theta_{y,t,z}$ (accounting for the generation conversion efficiency, $\eta^{down}_{y,z}$, plus the hourly inflows into the reservoir (equal to the installed reservoir capacity times the hourly inflow parameter $\rho^{max}_{y,t,z}$). Eq. 71 represents the energy level of the reservoir at the first hour of the simulation, calculated using the initial energy level factor, $w^{level}_{y,z}$, and the energy capacity of the reservoir. The maximum energy level at any given time t must then be less than or equal to the total reservoir capacity, as shown in Eq. 72. Electricity production is constrained by either the the net installed capacity (accounting for the generation efficiency, see Eq. 73) or by the current energy level in the reservoir (see Eq. 74), whichever is more binding. In addition, electricity production from hydro resources can be constrained to always be above a minimum output parameter $\rho^{min}_{y,z}$ to represent operational constraints related to minimum streamflows or other demands for water from hydro reservoirs.

$$\Gamma_{y,t,z} = \Gamma_{y,t-1,z} - \left(\frac{\Theta_{y,t,z}}{\eta^{down}_{y,z}}\right) + \left(\frac{\rho^{max}_{y,t,z} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\mu^{stor}_{y,z}}\right), \qquad \forall y \in \mathcal{W}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \quad (70)$$

$$\Gamma_{y,1,z} = \frac{w^{level}_{y,z} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\mu^{stor}_{y,z}}, \qquad \forall y \in \mathcal{W}, \forall z \in \mathcal{Z} \quad (71)$$

$$\Gamma_{y,t,z} \leq \frac{(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\mu^{stor}_{y,z}}, \qquad \forall y \in \mathcal{W}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \quad (72)$$

$$\Theta_{y,t,z} \leq \eta_{y,z}^{down}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{W}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \qquad (73)$$

$$\Theta_{y,t,z} \leq \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{W}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \qquad (74)$$

$$\Theta_{y,t,z} \geq \rho_{y,z}^{min}, \qquad \forall y \in \mathcal{W}, z \in \mathcal{Z} \qquad (75)$$

5.16 Accounting for Operating Reserves

If GenX is configured to require operating reserves, a set of additional constraints are added and several constraints for each generating and storage resource type capable of providing reserves are replaced by alternative versions incorporating reserve provision.

As currently implemented, GenX can model requirements for two types of operating reserves: frequency regulation or primary reserves; and secondary reserves, also known as balancing energy, spinning reserves, or contingency reserves. Reserve requirements and technical definitions vary across system operators, so there is no universal definition of reserve categories. Requirements for the two reserve categories and constraints on the ability of different resources to provide each reserve class can be parameterized to reflect different practices according to user preference.

However, as GenX is currently written, the primary reserve or frequency regulation requirement is intended to represent system operator requirements for automatic inertial response to deviations in frequency out of the nominal operating range (i.e., 60 Hz in the United States, 50 Hz in Europe, etc.), typically achieved via the governor response of online generators with rotational mass (e.g., thermal generators, hydro generators, kinetic storage devices). It is also technically possible for inverter-connected devices (e.g., solar PV, electrochemical storage, fuel cells) and wind turbines to provide synthetic inertial response, although this is not common practice in most systems today. GenX is flexible to allow the user to specify the fraction of a given resource's capacity can be committed to frequency regulation/primary reserves, including asynchronous generators capable of providing synthetic inertia, if desired.

Secondary or spinning reserve requirements are intended to represent online (or certain quick start) generators capable of automatically and rapidly responding to system operator dispatch signals in case of a significant frequency imbalance or contingency (e.g., generator or transmission outage), typically within a 5-30 minute response time. The user can also specify the portion of capacity that can be committed to secondary reserves for each resource type, as desired.

Note that GenX does not currently include requirements for tertiary or replacement reserves (e.g., offline generators capable of starting up to replace secondary reserves after a contingency).

5.16.1 Operating Reserve Requirements

Eq. 76 and Eq. 77 require that in every hour the sum of primary reserves/frequency regulation provided across all technologies and zones must be greater than or equal to the system's reserve requirement. The hourly requirements for frequency regulation reserve are symmetric—equal in magnitude for regulation up and down—and are determined as a fraction of the total hourly demand and the total hourly generation from VRE resources, depending on system requirements for demand and VRE generation, F^D and F^{VRE}, respectively. This requirement thus represents regulation required to respond rapidly to forecast errors in demand or VRE output.

$$F^D \sum_{z \in \mathcal{Z}} D_{t,z} + F^{VRE} \sum_{z \in \mathcal{Z}} \sum_{y \in (\mathcal{D} \cup \mathcal{ND})} (\Omega_{y,z} \times \rho^{max}_{y,t,z}) \leq \sum_{y \in \mathcal{G}} \sum_{z \in \mathcal{Z}} f^{+}_{y,z,t}, \qquad \forall t \in \mathcal{T} \qquad (76)$$

$$F^D \sum_{z \in \mathcal{Z}} D_{t,z} + F^{VRE} \sum_{z \in \mathcal{Z}} \sum_{y \in (\mathcal{D} \cup \mathcal{ND})} (\Omega_{y,z} \times \rho^{max}_{y,t,z}) \leq \sum_{y \in \mathcal{G}} \sum_{z \in \mathcal{Z}} f^{-}_{y,z,t}, \qquad \forall t \in \mathcal{T} \qquad (77)$$

Secondary reserves requirements can be specified asymmetrically in the upward and downward directions as a function of demand, R^{+D} and R^{-D}, and VRE generation, R^{+VRE} and R^{-VRE}, for each hour. As shown in Eq. 78, the total secondary reserves up provided across all technologies and zones must be greater than or equal to the system requirement, unless the unmet reserve variable, $r^{+,unmet}_t$, takes a value different from zero, which entails a cost of π^{unmet} for each MW of unmet reserves. This penalty, π^{unmet}, should represent the increased probability of incurring costs of non-served energy as secondary reserves are depleted and is specified by the user.

As Eq. 78 illustrates, secondary reserve requirements up not only depend on demand and VRE generation, but also on a contingency requirement, $\alpha_t(\cdot)$, that changes depending on the model configuration, and is intended to represent the secondary reserves required to ramp up quickly in case of unanticipated generator or transmission line outages. As shown in Eq. 79 the contingency requirement can take one of four values: (Configuration 1) the capacity of the largest single generator that *could* be deployed in the system; (Configuration 2) the capacity of either the largest generator that *could* be deployed in the system or the largest transmission capacity in the system, whichever is greater; (Configuration 3) the capacity of either the largest generator that *has* been deployed in the system or the largest transmission capacity in the system, whichever is greater; or (Configuration 4) the capacity of the largest generator that has been *committed* in the specific hour in the system or the largest transmission capacity in the system, whichever is greater. The latter options for specifying contingency requirements based on actual generator investment or commitment decisions are more dynamic and realistic, but they entail greater computational burden due to the requirement for auxiliary variables and constraints required to track the largest generator built or committed. Implementing the *max* function for Configuration 3 adds a number of auxiliary constraints equal to the number of resource types eligible for construction, while Configuration 4 adds a number of auxiliary constraints equal to the number of resource types eligible for construction times the number of time steps modeled. Configuration 1 and 2 add only a single constraint and are thus simpler and more computationally efficient options, if desired.

$$R^{+D} \sum_{z \in \mathcal{Z}} D_{t,z} + R^{+VRE} \sum_{z \in \mathcal{Z}} \sum_{y \in (\mathcal{D} \cup \mathcal{ND})} (\Omega_{y,z} \times \rho^{max}_{y,t,z}) + \alpha_t(\cdot) \leq \sum_{y \in \mathcal{G}} \sum_{z \in \mathcal{Z}} r^{+}_{y,z,t} + r^{+,unmet}_t, \qquad \forall t \in \mathcal{T} \qquad (78)$$

$$
\alpha_t(\cdot) = \begin{cases} \max(\Omega^{size}_{y,z}) & \text{if cont. 1} \\ \\ \max(\max(\Omega^{size}_{y,z}), \max(\varphi^{max}_l)) & \text{if cont. 2} \\ \\ \max(\max(\Omega^{size}_{y,z}), \max(\varphi^{max}_l)) & \forall y \mid \Omega_{y,z} \neq 0 \text{ if cont. 3} \\ \\ \max(\max(\Omega^{size}_{y,z}), \max(\varphi^{max}_l)) & \forall y \mid \upsilon_{y,t,z} \neq 0 \text{ if cont. 4} \end{cases}, \quad \forall t \in \mathcal{T} \quad (79)
$$

Secondary reserves in the downward direction are a function solely of forecasted demand (R^{-D}) and VRE output ((R^{-VRE}). Total secondary reserves down provided across all technologies and zones must be greater than or equal to the system requirement, unless the unmet reserve variable, $r^{-,unmet}_t$, takes a value different from zero, incurring a cost of π^{unmet} for each MW of unmet reserves.

$$
R^{-D} \sum_{z \in \mathcal{Z}} D_{t,z} + R^{-VRE} \sum_{z \in \mathcal{Z}} \sum_{y \in (\mathcal{D} \cup \mathcal{ND})} (\Omega_{y,z} \times \rho^{max}_{y,t,z}) \leq \sum_{y \in \mathcal{G}} \sum_{z \in \mathcal{Z}} r^{-}_{y,z,t} + r^{-,unmet}_t, \quad \forall t \in \mathcal{T} \quad (80)
$$

5.16.2 Constraints on Contribution of Capacity to Reserves

If reserve requirements are modeled, an additional set of constraints is added for each resource type capable of providing reserves, as follows.

For thermal technologies to which discrete unit commitment decisions apply ($y \in (\mathcal{UC} \cap \mathcal{H})$), the maximum capacity that can be committed to reserves corresponds to a cluster-specific maximum reserve contribution parameter (expressed as a fraction of capacity) times the total committed capacity of the cluster in each time step. Distinct parameters and constraints apply to contribution of capacity to each class of reserves (primary and secondary) and in each direction (upwards and downwards).

$$
f^{+}_{y,z,t} \leq \iota^{+}_{y,z}(\Omega^{size}_{y,z} \times \upsilon_{y,t,z}), \quad \forall y \in (\mathcal{UC} \cap \mathcal{H}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \quad (81)
$$

$$
f^{-}_{y,z,t} \leq \iota^{-}_{y,z}(\Omega^{size}_{y,z} \times \upsilon_{y,t,z}), \quad \forall y \in (\mathcal{UC} \cap \mathcal{H}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \quad (82)
$$

$$
r^{+}_{y,z,t} \leq \gamma^{+}_{y,z}(\Omega^{size}_{y,z} \times \upsilon_{y,t,z}), \quad \forall y \in (\mathcal{UC} \cap \mathcal{H}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \quad (83)
$$

$$
r^{-}_{y,z,t} \leq \gamma^{-}_{y,z}(\Omega^{size}_{y,z} \times \upsilon_{y,t,z}), \quad \forall y \in (\mathcal{UC} \cap \mathcal{H}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \quad (84)
$$

For all other (non-clustered) technologies technologies that can provide reserves (including dispatchable renewables, energy storage devices, hydro reservoir resources, and thermal technologies to which discrete unit commitment decisions/constraints are not applied), the resources can contribute up to a maximum specified by a resource-specific maximum reserve contribution parameter times the hourly maximum power output for that resource. Note that non-dispatchable renewables ($\mathcal{ND}$) and price-responsive demand curtailment (or demand response, $\mathcal{DR}$) cannot provide reserves.

$$
f^{+}_{y,z,t} \leq \iota^{+}_{y,z} \times \rho^{max}_{y,t,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \quad \forall y \notin (\mathcal{UC} \cup \mathcal{ND} \cup \mathcal{DR}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \quad (85)
$$

$$f^-_{y,z,t} \leq \iota^-_{y,z} \times \rho^{max}_{y,t,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \notin (\mathcal{UC} \cup \mathcal{ND} \cup \mathcal{DR}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{86}$$
$$r^+_{y,z,t} \leq \gamma^+_{y,z} \times \rho^{max}_{y,t,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \notin (\mathcal{UC} \cup \mathcal{ND} \cup \mathcal{DR}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{87}$$
$$r^-_{y,z,t} \leq \gamma^-_{y,z} \times \rho^{max}_{y,t,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \notin (\mathcal{UC} \cup \mathcal{ND} \cup \mathcal{DR}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{88}$$

Finally, storage technologies can provide reserves when both charging and discharging by adjusting the amount of power injection or withdrawal according to system operator dispatch. The total amount of primary reserves and secondary reserves provided by storage technologies thus equals the sum provided by each process, charging (C) and discharging (D) (see Eq. 89 to Eq. 92).

$$f^+_{y,z,t} = f^{+D}_{y,z,t} + f^{+C}_{y,z,t}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{89}$$
$$f^-_{y,z,t} = f^{-D}_{y,z,t} + f^{-C}_{y,z,t}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{90}$$
$$r^+_{y,z,t} = r^{+D}_{y,z,t} + r^{+C}_{y,z,t}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{91}$$
$$r^-_{y,z,t} = r^{-D}_{y,z,t} + r^{-C}_{y,z,t}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{92}$$

5.16.3 Alternative Operating Constraints Applied if Reserves are Modeled

If reserves are modeled, the minimum and maximum power output constraints for each resource type capable of providing reserves are also replaced by alternate operating constraints described in the following subsections.

For thermal units to which unit commitment decisions apply, Eq. 34 and 35 are replaced by Eq. 93 and Eq. 94 respectively. As shown, the power output minus reserves down being provided by cluster y must stay above the minimum stable output of the committed units in the cluster at hour t. At the same time, the power output plus reserves up being provided by cluster y must stay below the maximum power output of the committed units in the cluster at hour t.

$$\Theta_{y,t,z} - f^-_{y,z,t} - r^-_{y,z,t} \geq \rho^{min}_{y,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{93}$$
$$\Theta_{y,t,z} + f^+_{y,z,t} + r^+_{y,z,t} \leq \rho^{max}_{y,t,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{94}$$

For other thermal generators, Eq. 42 and Eq. 43 for minimum and maximum power output are replaced by Eq. 95 and Eq. 96 respectively. As shown, the power output minus reserves down being provided by technology y must stay above the minimum stable output of the existing capacity. At the same time, the power output plus reserves up being provided by technology y must stay below the maximum power output of the existing capacity.

$$\Theta_{y,t,z} - f^-_{y,z,t} - r^-_{y,z,t} \geq \rho^{min}_{y,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{H}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{95}$$
$$\Theta_{y,t,z} + f^+_{y,z,t} + r^+_{y,z,t} \leq \rho^{max}_{y,t,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{H}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{96}$$

Dispatchable renewable resources may be able to provide reserves, if specified by the user (e.g., if $\iota^+_{y,z}$, $\iota^-_{y,z}$, $\gamma^+_{y,z}$, or $\gamma^+_{y,z} > 0$). Thus Eq. 44 presented in section 5.11 must now account for this ability by being replaced by Eq. 98. That is, for hours when there is curtailment of renewables, these resources can provide reserves up to the system. Additionally, Eq. 97 must be added in order to account for the ability of renewable resources to provide reserves down through curtailment. Non-dispatchable renewable resources (e.g., rooftop solar) are assumed not to have the ability to provide reserves, as they cannot respond to system operator dispatch.

$$\Theta_{y,t,z} - f^{-}_{y,z,t} - r^{-}_{y,z,t} \geq 0, \qquad \forall y \in \mathcal{D}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{97}$$

$$\Theta_{y,t,z} + f^{+}_{y,z,t} + r^{+}_{y,z,t} \leq \rho^{max}_{y,t,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{D}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{98}$$

Storage resources can provide reserves by either charging or discharging, which requires replacing the operational constraints presented in section 5.12. Providing reserves down while charging requires storage to increase the charging rate, thus increasing total system demand. Thus as shown by Eq. 99 the charging power plus the reserves down provided must remain below the maximum capacity, replacing Eq. 48. Additionally, charging power plus reserves down being provided must be less than or equal to the remaining storage capacity. Thus Eq. 100 replaces Eq. 49. Storage can also provide reserves up while charging by decreasing the charing rate and thus decreasing system demand. Eq. 101 must be implemented to ensure that total reserves up cannot exceed the current charging rate.

$$\Pi_{y,t,z} + f^{-C}_{y,z,t} + r^{-C}_{y,z,t} \leq \frac{(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\eta^{up}_{y,z}}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{99}$$

$$\Pi_{y,t,z} + f^{-C}_{y,z,t} + r^{-C}_{y,z,t} \leq \frac{(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\mu^{stor}_{y,z}} - \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{100}$$

$$\Pi_{y,t,z} - f^{+C}_{y,z,t} - r^{+C}_{y,z,t} \geq 0, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{101}$$

Likewise, storage can provide reserves up while discharging by increasing the discharging rate and thus producing more electricity. Eq. 102 ensures that power discharge plus total commitments to reserves up remain below the maximum discharge capacity, replacing Eq. 50. At the same time, discharging plus reserves up cannot be greater than the current level of stored energy as shown in Eq. 103 (replacing Eq. 51). Eq. 104 must also be implemented to allow for discharging to provide reserves down by reducing electricity production, and ensures that total reserves down does not exceed the current discharge rate.

$$\Theta_{y,t,z} + f^{+D}_{y,z,t} + r^{+D}_{y,z,t} \leq \eta^{down}_{y,z} \times (\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{102}$$

$$\Theta_{y,t,z} + f^{+D}_{y,z,t} + r^{+D}_{y,z,t} \leq \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{103}$$

$$\Theta_{y,t,z} - f^{-D}_{y,z,t} + r^{-D}_{y,z,t} \geq 0, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{104}$$

Additionally, Eq. 52 must be replaced by Eq. 105 to ensure that the total contribution of storage capacity to charging, discharging, and reserves in both directions does not exceed total installed capacity (accounting for single-trip charging and discharging efficiencies).

$$\left(\frac{\Theta_{y,t,z} + f^{+D}_{y,z,t} + r^{+D}_{y,z,t}}{\eta^{down}_{y,z}}\right) + \left(\eta^{up}_{y,z}(\Pi_{y,t,z} + f^{-C}_{y,z,t} + r^{-C}_{y,z,t})\right) \leq$$
$$(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \qquad (105)$$

Finally, hydro reservoir resources can provide reserves up and down while discharging just like storage systems do. Eq. 106 ensures that generation plus reserves up being provided remains below the maximum generation capacity, replacing Eq. 73. At the same time, generation plus reserves up cannot be greater than the current level of stored energy as shown in Eq. 107 (replacing Eq. 74). Eq. 108 must be implemented to allow for generation to provide reserves down—by reducing electricity production—ensuring that the reserves down being provided do not exceed the current power output.

$$\Theta_{y,t,z} + f^{+}_{y,z,t} + r^{+}_{y,z,t} \leq \eta^{down}_{y,z}(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z}), \qquad \forall y \in \mathcal{W}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \qquad (106)$$
$$\Theta_{y,t,z} + f^{+}_{y,z,t} + r^{+}_{y,z,t} \leq \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{W}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \qquad (107)$$
$$\Theta_{y,t,z} - f^{-}_{y,z,t} - r^{-}_{y,z,t} \geq 0, \qquad \forall y \in \mathcal{W}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \qquad (108)$$

5.16.4 Reserve Contributions from NACC and Heat Storage

The maximum amount of reserves that can be provided by clustered NACC technologies correspond to the cluster type specific maximum reserve contribution times the total peaking capacity of the committed capacity of the cluster. Base capacity of the NACC systems is assumed to operate at constant power output, unless not committed, and thus is not able to provide reserves.

$$f^{+}_{y,z,t} \leq \iota^{+}_{y,z}(\eta^{heat}_{y,z} \times \mu^{heat}_{y,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}), \qquad \forall y \in (\mathcal{UC} \cap \mathcal{AN}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \qquad (109)$$
$$f^{-}_{y,z,t} \leq \iota^{-}_{y,z}(\eta^{heat}_{y,z} \times \mu^{heat}_{y,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}), \qquad \forall y \in (\mathcal{UC} \cap \mathcal{AN}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \qquad (110)$$
$$r^{+}_{y,z,t} \leq \gamma^{+}_{y,z}(\eta^{heat}_{y,z} \times \mu^{heat}_{y,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}), \qquad \forall y \in (\mathcal{UC} \cap \mathcal{AN}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \qquad (111)$$
$$r^{-}_{y,z,t} \leq \gamma^{-}_{y,z}(\eta^{heat}_{y,z} \times \mu^{heat}_{y,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}), \qquad \forall y \in (\mathcal{UC} \cap \mathcal{AN}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \qquad (112)$$

Eq. 113 must be implemented to ensure that the amount of reserves down being provided by NACC peaking capacity is less than or equal to the current peaking heat being used at the units. Eq. 114 replaces Eq. 60 presented in section 5.14 to take into account reserves up being provided by peaking capacity of NACC systems. This constraint ensures that reserves up plus current heat being used for peaking generation does not exceed the peaking capacity of the committed units in the cluster.

$$\sigma_{x,t,z} + \nu_{y,t,z} - \frac{(f^{-}_{y,z,t} + r^{-}_{y,z,t})}{\eta^{heat}_{y,z}} \geq 0, \qquad \forall y \in (\mathcal{UC} \cap \mathcal{AN}), \forall x \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \qquad (113)$$
$$\sigma_{x,t,z} + \nu_{y,t,z} + \frac{(f^{+}_{y,z,t} + r^{+}_{y,z,t})}{\eta^{heat}_{y,z}}$$
$$\leq \mu^{heat}_{y,z} \times \Omega^{size}_{y,z} \times \upsilon_{y,t,z}, \qquad \forall y \in (\mathcal{UC} \cap \mathcal{AN}), \forall x \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \qquad (114)$$

Finally, resistance-heated heat storage can provide reserves up and down while charging, by changing the rate of charge (e.g., conversion of electricity to heat), just like other storage systems. Eq. 115 and 116 replace Eq. 63 and Eq. 64 to take into account for the ability to provide reserves down. Reserves down for the charging process require heat storage to increase the charging rate, thus increasing system demand. As shown by Eq. 115, the charging power plus the reserves down provided must remain below the maximum capacity. Additionally, charging power plus reserves down being provided must be less than or equal to the remaining storage capacity as shown in Eq. 116. In addition, Eq.117 must be implemented to ensure that no more reserves up than the current charging rate are provided. Note that unlike other storage devices, resistance-heated heat storage cannot provide reserves while *discharging*, since discharged heat has no direct effect on the power system. Resistance-heated heat storage can thus only provide reserves indirectly by contribution to NACC peaking cycles, as described above.

$$\Pi_{y,t,z} + f^{-}_{y,z,t} + r^{-}_{y,z,t} \leq \frac{(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\eta^{up}_{y,z}}, \qquad \forall y \in \mathcal{OH}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{115}$$

$$\Pi_{y,t,z} + f^{-}_{y,z,t} + r^{-}_{y,z,t} \leq \frac{(\overline{\Delta_{y,z}} + \Omega_{y,z} - \Delta_{y,z})}{\mu^{stor}_{y,z}} - \Gamma_{y,t,z}, \qquad \forall y \in \mathcal{OH}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{116}$$

$$\Pi_{y,t,z} - f^{+}_{y,z,t} - r^{+}_{y,z,t} \geq 0, \qquad \forall y \in \mathcal{OH}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{117}$$

5.17 Accounting for Distribution Losses and Expansion

If desired, GenX can represent distribution networks using a zonal approximation, with each zone $z \in \mathcal{V}$ representative of a different distribution network topology and voltage level. As electricity demand as well as capacity investment and operational decisions are indexed across each zone in the system, demand can be assigned to different distribution zones and certain distributed energy resources (DERs, such as distributed solar PV, energy storage, fuel cells, etc.) can be made eligible for installation and operation in each distribution zone.

Distribution zone topology is specified just as it is for transmission zones, using the $\varphi^{map}_{l,z}$ parameters, and power flows between distribution zones can be constrained using Eq. 9–11, to represent limits on aggregate transformer capacity between voltage levels.

GenX applies a novel approach to represent aggregate losses from power flows *within* distribution network zones as a segmentwise linear approximation of a polynomial function of both power injections and withdrawals within each zone. Based on detailed offline AC power flow simulations of large-scale realistic distribution networks [39], within zone distribution losses can be closely approximated as a polynomial function with a quadratic term for the absolute value of aggregate net power withdrawals (e.g., aggregate withdrawals less aggregate injections) within each zone, linear terms for both aggregate withdrawal and aggregate injection alone, and an intercept (e.g., $\ell_{z,t} = \varphi^{Net}_{z}|W_{z,t} - I_{z,t}|^2 + \varphi^{W}_{z}W_{z,t} + \varphi^{I}_{z}I_{z,t} + \varphi^{Int}_{z}$). GenX thus implements a segmentwise linear approximation of the quadratic term in this function plus the linear and constant terms, to represent within zone losses as per Eq. 118. The slopes and constraints on the lengths of each segment used for linear approximation of the quadratic term is calculated as per Eq. 118–120 to minimize error at any point in the domains $[0 : (\lambda^{I}_{z} + \overline{\triangle\lambda^{I}_{z}})]$ and $[0 : (\lambda^{W}_{z} + \overline{\triangle\lambda^{W}_{z}})]$ (e.g., over the realm of all possible injections and withdrawals that may be modeled). This method is based on the segmentwise linear interpolation of quadratic functions in [37].

$$
\begin{aligned}
&\ell_{z,t} = \varphi_z^{Net} \sum_{m=1}^{M} \left(S_{m,z}^{+} \times \S_{m,z,t}^{+} + S_{m,z}^{+} \times \S_{m,z,t}^{-} \right) \\
&\quad + \varphi_z^{W} \times W_{z,t} + \varphi_z^{I} \times I_{z,t} + \varphi_z^{Int}, && \forall z \in \mathcal{V}, \forall t \in \mathcal{T} \\
&\text{Where:} \\
&S_{m,z}^{+} = \frac{2 + 4 \times \sqrt{2} \times (m-1)}{1 + \sqrt{2} \times (2 \times M - 1)} (\lambda_z^I + \overline{\triangle \lambda_z^I}) && \forall m \in [1\colon M], z \in \mathcal{V} \\
&S_{m,z}^{-} = \frac{2 + 4 \times \sqrt{2} \times (m-1)}{1 + \sqrt{2} \times (2 \times M - 1)} (\lambda_z^W + \overline{\triangle \lambda_z^W}) && \forall m \in [1\colon M], z \in \mathcal{V} \\
&W_{z,t} = \sum_{d \in \mathcal{V}} \varphi_{z,d}^{down} \times \left(D_{z,t} + \sum_{y \in \mathcal{O}, \mathcal{HO}} (\Pi_{y,t,z}) + \sum_{y \in \mathcal{DR}} (\Theta_{y,t,z}) \right), && \forall z \in \mathcal{Z}, t \in \mathcal{T} \\
&I_{z,t} = \sum_{y \notin \mathcal{DR}} (\Theta_{y,t,z}) && \forall z \in \mathcal{Z}, t \in \mathcal{T}
\end{aligned}
\tag{118}
$$

$$
\begin{aligned}
&\S_{m,z,t}^{+}, \leq \overline{\S_{m,z}^{+}}, && \forall m \in [1\colon M], z \in \mathcal{Z}, t \in \mathcal{T} \\
&\text{Where:} \\
&\overline{\S_{m,z}^{+}} = \begin{cases} \frac{(1+\sqrt{2})}{1+\sqrt{2}\times(2\times M-1)} (\lambda_z^I + \overline{\triangle \lambda_z^I}) & \text{if m=1} \\ \\ \frac{2\times\sqrt{2}}{1+\sqrt{2}\times(2\times M-1)} (\lambda_z^I + \overline{\triangle \lambda_z^I}) & \text{if m} > 1 \end{cases}
\end{aligned}
\tag{119}
$$

$$
\begin{aligned}
&\S_{m,z,t}^{-}, \leq \overline{\S_{m,z}^{-}}, && \forall m \in [1\colon M], z \in \mathcal{Z}, t \in \mathcal{T} \\
&\text{Where:} \\
&\overline{\S_{m,z}^{-}} = \begin{cases} \frac{(1+\sqrt{2})}{1+\sqrt{2}\times(2\times M-1)} (\lambda_z^W + \overline{\triangle \lambda_z^W}) & \text{if m=1} \\ \\ \frac{2\times\sqrt{2}}{1+\sqrt{2}\times(2\times M-1)} (\lambda_z^W + \overline{\triangle \lambda_z^W}) & \text{if m} > 1 \end{cases},
\end{aligned}
\tag{120}
$$

Eq. 121–122 implement the linearization of the absolute value of net withdrawal ($|W_{z,t} - I_{z,t}|$). If the net aggregate power withdrawal in zone z at time t is in the positive domain ($(W_{z,t} - I_{z,t}) > 0$), then the values of the positive domain piecewise segments $\S_{m,z,t}^{+}\, \forall m \geq 1$ must sum to the total net withdrawal. At the same time, this pair of constraints then ensures $\S_{m,z,t}^{-} = 0\, \forall m \geq 1$ and that a “zero segment” $\S_{0,z,t}^{-}$ equals the net withdrawal. This zero segment is ignored in calculating losses in Eq 118, ensuring that losses are solely a function of the positive portion of the absolute value. In contrast, if net aggregate power withdrawal in zone z at time t is in the negative domain ($(W_{z,t} - I_{z,t}) < 0$), then the values of the negative domain piecewise segments $\S_{1,z,t}^{-} : \S_{M,z,t}^{-}$ must sum to the total (negative) net withdrawal. The positive domain segments $\S_{0,z,t}^{+} = 0\, \forall m \geq 1$ and that the positive domain “zero segment” $\S_{0,z,t}^{+}$ equals the net withdrawal (and is likewise ignored in losses calculations).

$$\sum_{m=1}^{M}\left(\S^{+}_{m,z,t}\right) - \S^{+}_{0,z,t} = (W_{z,t} - I_{z,t}) \qquad \forall z \in \mathcal{V}, \forall t \in \mathcal{T} \tag{121}$$

$$\sum_{m=1}^{M}\left(\S^{-}_{m,z,t}\right) - \S^{-}_{0,z,t} = -(W_{z,t} - I_{z,t}) \qquad \forall z \in \mathcal{V}, \forall t \in \mathcal{T} \tag{122}$$

If integer unit commitment decisions are modeled, then Eq. 118–122 allow the solver to produce "phantom losses," or losses unrelated to physical power withdrawals or injections in each distribution zone by filling segments of the piecewise approximation out of order (e.g., using segments associated with larger slopes in Eq. 118 to represent the absolute value of net withdrawals in Eq. 121–122). In some circumstances, increasing total demand by creating phantom losses may avoid a costly discrete start-up or shut-down decision and reduce the total objective function, despite increasing variable costs associated with supplying the additional increment of demand (see [38]). To prevent phantom losses, Eq 123–128 are added to the model whenever discrete unit commitment decisions are modeled. Eq. 123–124 require that each segment in the linear approximation can be nonzero only if the binary auxiliary variable ($ON^{+}_{m,z,t}$ or $ON^{-}_{m,z,t}$) for that segment equals 1. Then Eq 125–126 require that if any segment $m+1$ is nonzero, the proceeding segment m must take on its maximum possible value, where the maximum length per segment ($\overline{\S^{+}_{m,z}}$ or $\overline{\S^{-}_{m,z}}$) is given by Eq. 119–120 above. Eq. 127–128 are binary constraints to deal with absolute value of net withdrawals. If net withdrawal is positive, $\S^{+}_{0,m,t}$ must be zero; if net withdrawal is negative, $\S^{+}_{0,z,t}$ must be positive and takes on value of the full net withdrawal, forcing all $\S^{+}_{m,z,t}$ other segments ($m \geq 1$) to be zero. Conversely, if the net withdrawal is negative, $\S^{-}_{0,z,t}$ must be zero; if net withdrawal is positive, $\S^{-}_{0,z,t}$ must be positive and takes on value of the full net withdrawal, forcing all $\S^{-}_{m,z,t}$ other segments ($m \geq 1$) to be zero.

$$\S^{+}_{m,z,t} \leq \overline{\S^{+}_{m,z}} \times ON^{+}_{m,z,t} \qquad \forall m \in [1\colon M], z \in \mathcal{V}, t \in \mathcal{T} \tag{123}$$

$$\S^{-}_{m,z,t} \leq \overline{\S^{-}_{m,z}} \times ON^{-}_{m,z,t} \qquad \forall m \in [1\colon M], z \in \mathcal{V}, t \in \mathcal{T} \tag{124}$$

$$\S^{+}_{m,z,t} \geq ON^{+}_{m+1,z,t} \times \overline{\S^{+}_{m,z}} \qquad \forall m \in [1\colon (M-1)], z \in \mathcal{V}, t \in \mathcal{T} \tag{125}$$

$$\S^{-}_{m,z,t} \geq ON^{-}_{m+1,z,t} \times \overline{\S^{-}_{m,z}} \qquad \forall m \in [1\colon (M-1)], z \in \mathcal{V}, t \in \mathcal{T} \tag{126}$$

$$\S^{+}_{0,z,t} \leq (\lambda^{W}_{z} + \overline{\triangle\lambda^{W}_{z}}) \times (1 - ON^{+}_{1,z,t}), \qquad \forall z \in \mathcal{V}, \forall t \in \mathcal{T} \tag{127}$$

$$\S^{-}_{0,z,t} \leq (\lambda^{I}_{z} + \overline{\triangle\lambda^{I}_{z}}) \times (1 - ON^{-}_{1,z,t}), \qquad \forall z \in \mathcal{V}, \forall t \in \mathcal{T} \tag{128}$$

Constraints on aggregate power withdrawals and injections in each distribution zone z are also applied, to represent the aggregate capacity of the network zone to absorb injections or deliver withdrawals without violating network operating constraints, such as voltage limits, as per Eq. 129–130. Note as aggregate peak power withdrawal and injection periods drive network capacity investment requirements, the user can specify a subset of time steps $\mathcal{PW}, \mathcal{PI} \subset \mathcal{T}$ during which aggregate peak withdrawals and injections are likely to occur. The distribution network withdrawal and injection constraints then only apply in these time periods to limit computational burden.

Eq. 129–130 specify that aggregate power withdrawals and injections (W and I, as given by Eq. 118

above) in each distribution zone z cannot exceed the sum of the specified initial network capacity ($\overline{\lambda_z^W}, \overline{\lambda_z^I}$) plus any network margin gained via investment in network expansion (e.g., reinforcements and upgrades, $\triangle\lambda_z^W, \triangle\lambda_z^I$) plus any network margin gained via optimal dispatch of distributed energy resources ($\phi_{z,t}^W, \phi_{z,t}^I$). Network capacity can only be expanded up to a specified maximum additional capacity for withdrawal and injection, as per 131–132, to reflect limits on the ability to reinforce or expand networks (if any).

$$\lambda_z^W + \triangle\lambda_z^W + \phi_{z,t}^W \geq W_{z,t}, \qquad \forall z \in \mathcal{V}, t \in \mathcal{PW} \tag{129}$$

$$\lambda_z^I + \triangle\lambda_z^I + \phi_{z,t}^I \geq I_{z,t}, \qquad \forall z \in \mathcal{V}, t \in \mathcal{PI} \tag{130}$$

$$\triangle\, \lambda_z^W \leq \overline{\triangle\lambda_z^W}, \qquad \forall z \in \mathcal{V} \tag{131}$$

$$\triangle\, \lambda_z^I \leq \overline{\triangle\lambda_z^I}, \qquad \forall z \in \mathcal{V} \tag{132}$$

The ability to capture the additional network margin (or ability to accommodate aggregate peak withdrawals or injections) achieved via optimal dispatch of distributed energy resources (DERs) and demand response, given by the variables $\phi_{z,t}^W, \phi_{z,t}^I$, is a novel feature of GenX. Where aggregate peak demand exceeds network capacity margins, distribution system operators may dispatch (directly or via aggregators or price signals) distributed resources within a given distribution network zone to inject power (e.g., via distributed generation or storage discharging) or reduce withdrawals (e.g., via demand response) in targeted locations to render network power flows feasible where they would otherwise violate network operating limits (such as voltage constraints or transformer thermal limits). Similarly, curtailment of injections by dispatchable distributed energy resources targeted to the right parts of the distribution network may also relieve network constraints and allow for an increase in overall injections elsewhere, reducing the need for network capacity upgrades (e.g., increasing DER hosting capacity). This additional network margin gained via DER operation thus acts as a substitute for the additional network investments that would otherwise be necessary to accommodate peak withdrawals or injections. In this manner, GenX can model "non-wires" alternatives or operational strategies to harness optimal dispatch of DERs and demand response to avoid investments in network "wires" solutions or physical upgrades to distribution networks with Eq. 133–136.

Based on detailed offline optimal AC power flow modeling of realistic large-scale distribution networks [40], this additional peak withdrawal margin gained by optimal dispatch of DER injection and demand curtailment can be closely approximated by a polynomial function with a square-root term and a linear term. This polynomial function is implemented with a segmentwise linear approximation of the square-root term as per Eq. 133–135. GenX automatically parameterizes the slope of each segment for the segmentwise linear approximation by using a function that fits M segments, each of steadily increasing length ($\overline{\S_{m,z}^{\phi,W}}$) specified by Eq. 135, in order to minimize the approximation error across the domain $[0 : \overline{\phi_z^W}]$.

$$\phi^{W}_{z,t} = N^{W,sqrt}_{z} \times \sum_{m=1}^{M} \left(S^{\phi,W}_{m,z} \times \S^{\phi,W}_{m,z,t} \right) + N^{W,linear}_{z} \times DER^{W}_{z,t} \qquad \forall z \in \mathcal{Z}, t \in \mathcal{PW}$$

Where:

$$DER^{W}_{z,t} = \sum_{d \in \mathcal{V}} \varphi^{down}_{z,d} \times \left(\sum_{y \notin \mathcal{ND},\mathcal{DR}} (\Theta_{y,t,z}) + \sum_{y \in DR} (\Pi_{y,t,z}) + \sum_{s \in S} (\Lambda_{s,t,z}) \right), \qquad \forall z \in \mathcal{Z}, t \in \mathcal{PW} \quad (133)$$

$$\sum_{m=1}^{M} \S^{\phi,W}_{m,z} = DER^{W}_{z,t} \qquad \forall z \in \mathcal{Z}, t \in \mathcal{PW} \quad (134)$$

$$\S^{\phi,W}_{m,z} \leq \overline{\S^{\phi,W}_{m,z}} \qquad \forall m \in [1:M], z \in \mathcal{Z}$$

Where:

$$\overline{\S^{\phi,W}_{m,z}} = \frac{6 \times \overline{\phi^{W}_{z}}}{M \times (M+1) \times (2 \times M + 1)} \times m^2 \qquad \forall m \in [1:M], z \in \mathcal{V} \quad (135)$$

$$\phi^{I}_{z,t} = 0, \qquad \forall z \in \mathcal{Z}, t \in \mathcal{T} \quad (136)$$

Note that in this version of the model, network injection margins gained via optimal dispatch of DERs is constrained to equal zero (Eq. 136) and thus not considered. At this stage, further research and offline modeling is required to determine an accurate representation for injection margins that may be gained by optimal curtailment of DERs, and whether this gain is supralinear. Results from this future work will be incorporated into subsequent versions of GenX. In the current version of GenX, dispatchable DERs may still reduce their net injections during aggregate peak injection periods to directly reduce the value of $\lambda^{I}_{z,t}$ and thus have a linear effect on the need for network reinforcement as associated costs in the objection function.

5.18 Non-Negativity and Integrality Constraints

A set of non-negativity and integrality constraints also apply as follows.

Eq. 137 ensures that investment decisions on new capacity and retirements for all technologies must be greater than or equal to zero for any technologies that are not subject to unit commitment constraints; while Eq. 138 ensures that investment decisions for units subject to unit commitment must take positive integer values, greater than or equal to zero.

$$\Omega_{y,z}, \; \Delta_{y,z} \geq 0, \qquad \forall y \notin \mathcal{UC}, \forall z \in \mathcal{Z} \quad (137)$$

$$\Omega_{y,z}, \; \Delta_{y,z} \geq 0 \in \mathbb{Z}_{+}, \qquad \forall y \in \mathcal{UC}, \forall z \in \mathcal{Z} \quad (138)$$

Eq. 139 ensures that power flow losses and power flow auxiliary variables are greater than or equal to zero. Eq. 142 ensures transmission expansion investments are greater than or equal to zero, while Eq. 143 forces transmission expansion to be zero for lines not eligible for expansion.

$$\ell_{l,t},\ \Phi^{+}_{l,t},\ \Phi^{-}_{l,t}, \geq 0, \qquad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{139}$$

$$\S^{+}_{m,l,t},\ \S^{-}_{m,l,t} \geq 0, \qquad \forall m \in \mathcal{M}, \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{140}$$

$$\triangle\, \Phi^{m,+,ON}_{l,t},\ \ \triangle\, \Phi^{m,-,ON}_{l,t} \in \{0,1\}, \qquad \forall m \in \mathcal{M}, \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{141}$$

$$\triangle\, \varphi^{max}_{l} \geq 0, \qquad \forall l \in \mathcal{E} \tag{142}$$

$$\triangle\, \varphi^{max}_{l} = 0, \qquad \forall l \notin \mathcal{E} \tag{143}$$

Eq. 144 ensures that power output must be greater than or equal to zero for all technologies, while Eq. 145 ensures that charging power must be greater than or equal to zero for all technologies with the ability to withdraw power from the grid ($y \in (\mathcal{O} \cup \mathcal{HO} \cup \mathcal{DR})$). Eq. 146 ensures that stored energy must be greater than or equal to zero for all technologies with the ability to store energy ($y \in (\mathcal{O} \cup \mathcal{HO} \cup \mathcal{DR} \cup \mathcal{W})$). Eq. 147–148 then forces the charging and storage variables for all other technologies to be zero.

$$\Theta_{y,t,z} \geq 0, \qquad \forall y \in \mathcal{G}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{144}$$

$$\Pi_{y,t,z} \geq 0, \qquad \forall y \in (\mathcal{O} \cup \mathcal{HO} \cup \mathcal{DR}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{145}$$

$$\Gamma_{y,t,z} \geq 0, \qquad \forall y \in (\mathcal{O} \cup \mathcal{HO} \cup \mathcal{DR} \cup \mathcal{W}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{146}$$

$$\Pi_{y,t,z} = 0, \qquad \forall y \notin (\mathcal{O} \cup \mathcal{HO} \cup \mathcal{DR}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{147}$$

$$\Gamma_{y,t,z} = 0, \qquad \forall y \notin (\mathcal{O} \cup \mathcal{HO} \cup \mathcal{DR} \cup \mathcal{W}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{148}$$

Eq. 149 restricts the amount of non-served energy of all different segments of consumers to be greater than or equal to zero.

$$\Lambda_{s,t,z} \geq 0, \qquad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{149}$$

Eq. 150 forces the unit commitment variables to be zero for all technologies to which unit commitment decisions do not apply, while Eq. 151 ensures the unit commitment variables are integer values greater than or equal to zero for all technologies subject to discrete unit commitment decisions.

$$\upsilon_{y,t,z},\ \chi_{y,t,z},\ \zeta_{y,t,z} = 0, \qquad \forall y \notin \mathcal{UC}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{150}$$

$$\upsilon_{y,t,z},\ \chi_{y,t,z},\ \zeta_{y,t,z} \geq 0 \in \mathbb{Z}_{+}, \qquad \forall y \in \mathcal{UC}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{151}$$

Eq. 153 ensures that the heat discharge variables from heat storage technologies are greater than or equal to zero, while Eq. 152 forces them to be exactly zero for non-heat storage technologies. Analogously, Eq. 155 and Eq. 154 ensure that heat coming from NG is greater than or equal to zero for NACC and exactly zero otherwise.

$$\epsilon_{y,t,z},\ \sigma_{y,t,z} = 0, \qquad \forall y \notin \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{152}$$

$$\epsilon_{y,t,z},\ \sigma_{y,t,z} \geq 0, \qquad \forall y \in \mathcal{HO}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{153}$$
$$\nu_{y,t,z} = 0, \qquad \forall y \notin \mathcal{AN}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{154}$$
$$\nu_{y,t,z} \geq 0, \qquad \forall y \in \mathcal{AN}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{155}$$

Eq. 156 ensures reserve-related variables are greater than or equal to zero, while Eq. 157 forces reserve-related variables to be zero for technologies that cannot provide reserves. In the same way, Eq. 158 ensures auxiliary reserve variable for storage are greater than or equal to zero, while Eq. 159 forces auxiliary reserve variable to be zero for non-storage technologies. Eq. 160 ensure that unmet reserves variables are greater than or equal to zero.

$$r^{+}_{y,z,t}, r^{-}_{y,z,t}, f^{+}_{y,z,t}, f^{-}_{y,z,t} \geq 0, \qquad \forall y \notin (\mathcal{ND} \cup \mathcal{DR}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{156}$$
$$r^{+}_{y,z,t}, r^{-}_{y,z,t}, f^{+}_{y,z,t}, f^{-}_{y,z,t} = 0, \qquad \forall y \in (\mathcal{ND} \cup \mathcal{DR}), \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{157}$$
$$r^{+C}_{y,z,t}, r^{-C}_{y,z,t}, f^{+C}_{y,z,t}, f^{-C}_{y,z,t}, r^{+D}_{y,z,t}, r^{-D}_{y,z,t}, f^{+D}_{y,z,t}, f^{-D}_{y,z,t} \geq 0, \qquad \forall y \in \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{158}$$
$$r^{+C}_{y,z,t}, r^{-C}_{y,z,t}, f^{+C}_{y,z,t}, f^{-C}_{y,z,t}, r^{+D}_{y,z,t}, r^{-D}_{y,z,t}, f^{+D}_{y,z,t}, f^{-D}_{y,z,t} = 0, \qquad \forall y \notin \mathcal{O}, \forall t \in \mathcal{T}, \forall z \in \mathcal{Z} \tag{159}$$
$$r^{+,unmet}_{t}, r^{-,unmet}_{t} \geq 0, \qquad \forall t \in \mathcal{T} \tag{160}$$

Eq. 161–162 ensure that total losses and each segment in the segmentwise approximation of the quadratic term for distribution losses are greater than or equal to zero.

$$\ell_{z,t} \geq 0 \qquad \forall z \in \mathcal{V}, t \in \mathcal{T} \tag{161}$$
$$\S^{+}_{m,z,t}, \S^{-}_{m,z,t} \geq 0, \quad \forall m \in [1:M], z \in \mathcal{V}, t \in \mathcal{T} \tag{162}$$

Likewise, Eq. 163–165 ensure that distribution network withdrawal and injection capacity expansions, network margin gained via dispatch of distributed resources, and the segments for the segmentwise approximation of the square-root term in the withdrawal margin gained function are all non-zero.

$$\triangle\lambda^{W}_{z}, \triangle\lambda^{I}_{z} \geq 0, \qquad \forall z \in \mathcal{V} \tag{163}$$
$$\phi^{W}_{z,t}, \phi^{I}_{z,t} \geq 0, \qquad \forall z \in \mathcal{V}, t \in \mathcal{T} \tag{164}$$
$$\S^{\phi,W}_{m,z,t} \geq 0, \qquad \forall m \in [1:M], z \in z\mathcal{V}, t \in \mathcal{T} \tag{165}$$

Note that all variables that are constrained above to be exactly equal to zero are removed from the model, along with related constraints, by the presolve step applied by the commercial solver (e.g., Gurobi), so these constraints are included above for completeness, but do not contribute to the computational dimensionality of the problem.

Appendix A: GenX Use Cases

This section provides uses cases of the GenX model, reflecting research to date that employs the tool. The section is intended to give the reader examples of the kinds of research and analysis that can be performed with the model, but by no means presents an exhaustive view of the highly-configurable model's full potential. We will update this section periodically.

Analyzing technological pathways to decarbonization of the power sector: The role of advanced nuclear reactors and thermal energy storage

In [9], N. Sepulveda employed the GenX model to explore the potential cost implications of several different technological pathways for decarbonization of the electricity generation mix in the year 2050. The model was used to explore the sensitivity of decarbonization costs to a wide range of different emissions limits and technology cost and availability assumptions. In particular, Sepulveda explores the potential for advanced nuclear reactor designs (e.g., the Nuclear Air-Brayton Cycle reactor concept) and new high-temperature thermal energy storage technologies to reduce the cost of decarbonization in the power sector. This work also marks the first use of the GenX model to explore interactions with industrial heat demand sector. In this work, GenX is configured to model a full year of hourly chronological variability, incorporates integer unit commitment decisions for thermal generators, and includes a wide range of generation, energy storage, and demand-side resources. The study employed a single zone representation of networks (e.g., network topography was ignored) and considered two locations with different renewable resource quality and demand patterns. More than 200 distinct runs were performed using a parallel supercomputing computing cluster.

Case study of distributed energy storage in the *MIT Energy Initiative Utility of the Future* study

In the first work demonstrating GenX's capabilities to consider distributed energy resources and impacts on distribution and transmission networks, J. Jenkins performed a case study on the role of distributed energy storage featured in the *MIT Energy Initiative Utility of the Future* study [45] (see Chapter 8). Battery energy storage can be deployed almost anywhere and at multiple scales, ranging from a few kW to the multi-MW range, and can provide multiple services to the power system, such as reserves, capacity, energy arbitrage, and firm capacity. Siting decisions for energy storage thus must weigh tradeoffs between economies of unit scale and locational value that can be captured by different size systems deployed at different voltage levels. In this case study, GenX was used to explore opportunities for using energy storage to avoid or defer transmission network reinforcement costs in a constrained link between a bulk power system generation zone and a distribution network zone where energy consumers are located. In the case study, storage can be employed at the 5 kW scale in low-voltage distribution or the 100 kW scale in medium-voltage distribution, in both cases "downstream" from the transmission constraint, which can reduce the need for transmission capacity expansion. Storage can also be employed at the 25 MW scale in the bulk transmission voltage zone "upstream" of the constraint and has a lower unit cost (e.g., $/kW) than distributed storage. The study uses a Spain-like test system and the model was configured to model a full year of hourly chronological variability. The convex hull of integer unit commitment constraints (e.g., linear relaxation of discrete unit commitment decisions) was used to accelerate solve time. A multi-zonal network framework was employed with a bulk power generation zone linked by a constrained transmission pathway to a distribution zone with two voltage levels. Transmission and distribution losses were considered, as well as transmission reinforcement options.

Case study for the OECD-NEA "Dealing with System Costs in Decarbonising Electricity Systems: Policy Options" study

In this work, GenX was used in the framework of an OECD-NEA study to assess the technical and economic impacts of different variable renewable energy (VRE) targets in a European region subject to a strong carbon constraint. N. Sepulveda performed techno-economic simulations using GenX representing the region as a two zone system subject to different VRE mandates using a full year of hourly chronological variability, incorporating a linear relaxation of unit commitment decisions, and considering a wide range of generation, energy storage, and demand-side resources. The study used a set of well-defined sensitivity analysis to define the impact of important parameters, including the availability of flexible hydro resources and the degree of transmission interconnections between neighboring regions. The final report is being produced at the OECD in Paris.

The role of flexible base resources in deep decarbonization of the power sector

In [46], N. Sepulveda, J. Jenkins and co-authors use GenX to perform the first systematic investigation of the role of low-carbon "flexible base" resources and their interaction with variable renewable resources, energy storage, and demand-side flexibility in achieving deep decarbonization of electric power systems. Flexible base resources are defined as a class of electricity generation resources economically suited to operating at high annual utilization rates (generally >50%), providing dispatchable power to meet system reliability needs, and adjusting output as renewable energy production fluctuates. The paper evaluates the potential role of both flexible nuclear power plants and gas plants with CCS as flexible base resources under various CO_2 emissions limits. This work evaluates 304 distinct cases covering significant uncertainty in future technology cost and availability, geographic differences in renewable resource availability and patterns of electricity demand, increasingly stringent emissions limits, and the impact of operating reserve requirements. In this work, GenX was configured to consider a full year of hourly chronological variability in a single zone problem. Integer unit commitment decisions were modeled for all thermal units, and a subset of runs employed frequency regulation and operating reserve requirements. Parallel supercomputing resources were employed to perform a wide range of cases covering key parametric uncertainties.

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